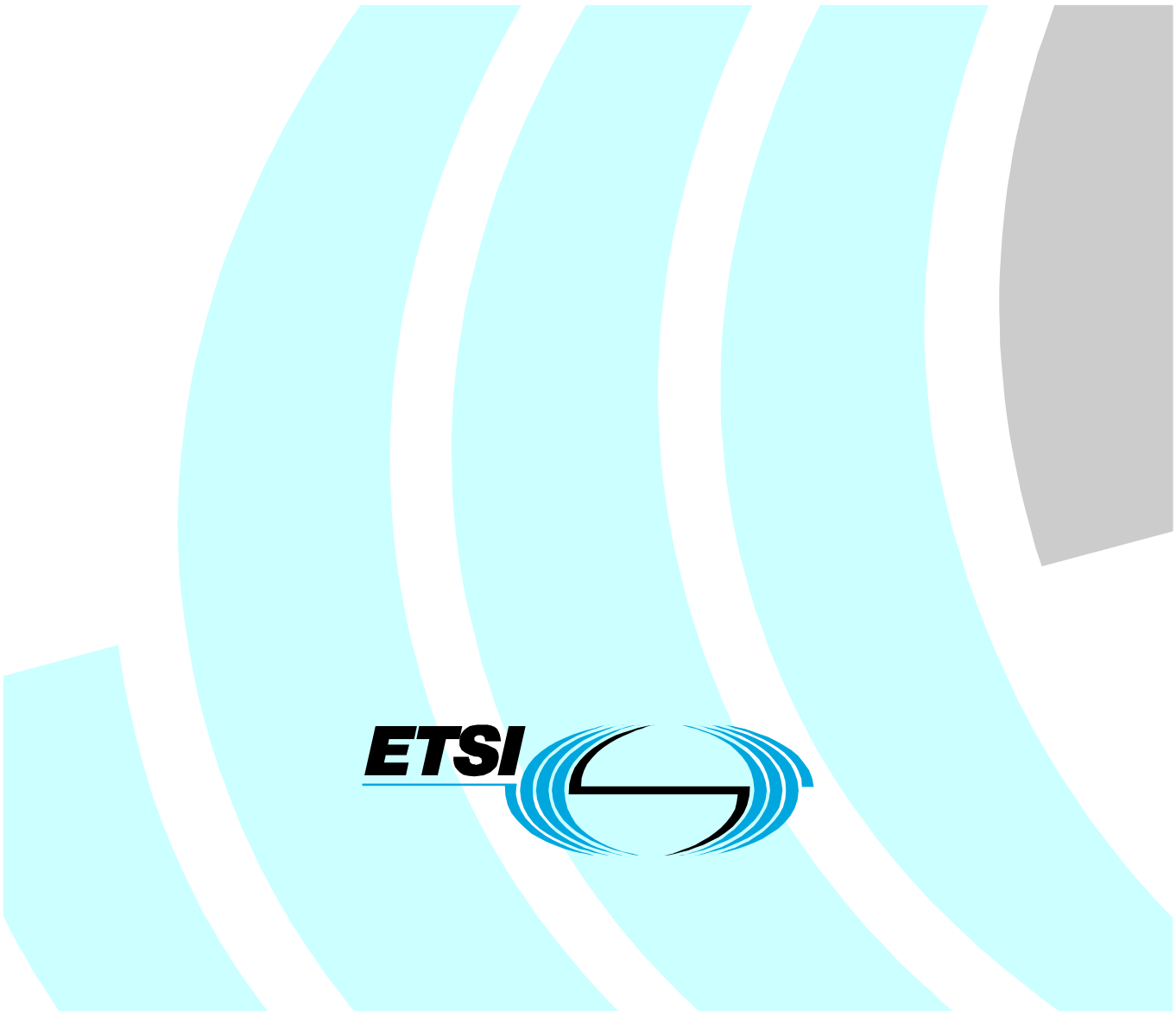
ETSI TS 102 250-6 V1.2.1 (2004-10)

Technical Specification

Speech Processing, Transmission and Quality Aspects (STQ); QoS aspects for popular services in GSM and 3G networks; Part 6: Post processing and statistical methods



Reference
RTS/STQ-00061m

Keywords
3G, GSM, network, QoS, service, speech

ETSI

650 Route des Lucioles
F-06921 Sophia Antipolis Cedex - FRANCE

Tel.: +33 4 92 94 42 00 Fax: +33 4 93 65 47 16

Siret N° 348 623 562 00017 - NAF 742 C
Association à but non lucratif enregistrée à la
Sous-Préfecture de Grasse (06) N° 7803/88

Contents

Intellectual Property Rights 6
Foreword 6
Introduction 7
1 Scope 8
2 References 8
3 Definitions, symbols and abbreviations 8
3.1 Definitions 8
3.2 Symbols 9
3.3 Abbreviations 9
4 Important measurement data types in mobile communications 9
4.1 Data with binary values 10
4.2 Data out of time-interval measurements 10
4.3 Measurement of data throughput 10
4.4 Data concerning quality measures 10
5 Distributions and moments 11
5.1 Introduction 11
5.2 Continuous and discrete distributions 12
5.3 Definition of density function and distribution function 12
5.3.1 Probability Distribution Function (PDF) 12
5.3.2 Cumulative Distribution Function (CDF) 13
5.4 Moments and quantiles 13
5.5 Estimation of moments and quantiles 15
5.6 Important distributions 16
5.6.1 Continuous distributions 16
5.6.1.1 Normal distribution 16
5.6.1.1.1 Standard normal distribution 17
5.6.1.1.2 Central limit theorem 18
5.6.1.1.3 Transformation to normality 18
5.6.1.2 Log-Normal distribution 18
5.6.1.2.1 Use-case: transformations 19
5.6.1.3 Exponential distribution 19
5.6.1.4 Weibull distribution 20
5.6.1.5 Pareto distribution 21
5.6.1.6 Extreme distribution (Fisher-Tippett distribution) 22
5.6.2 Testing distributions 22
5.6.2.1 Chi-Square distribution with *n* degrees of freedom 23
5.6.2.1.1 Further relations 24
5.6.2.1.2 Relation to empirical variance 24
5.6.2.2 Student t-distribution 24
5.6.2.2.1 Relation to normal distribution 25
5.6.2.3 F distribution 26
5.6.2.3.1 Quantiles 27
5.6.2.3.2 Approximation of quantiles 27
5.6.2.3.3 Relations to other distributions 28
5.6.3 Discrete distributions 28
5.6.3.1 Bernoulli distribution 28
5.6.3.2 Binomial distribution 29
5.6.3.3 Geometric distribution 30
5.6.3.4 Poisson distribution 31
5.6.4 Transitions between distributions and appropriate approximations 32
5.6.4.1 From binomial to Poisson distribution 32
5.6.4.2 From binomial to Normal distribution 32
5.6.4.3 From Poisson to Normal distribution 32

5.6.5 Truncated distributions 33
5.6.6 Distribution selection and parameter estimation 33
5.6.6.1 Test procedures 33
5.6.6.1.1 Chi-Square test 33
5.6.6.1.2 Kolmogorov-Smirnov test 33
5.6.6.1.3 Shapiro-Wilk test 33
5.6.6.2 Parameter estimation methods 34
5.7 Evaluation of measurement data 34
5.7.1 Statistical tests 34
5.7.1.1 Formulation of statistical tests 34
5.7.1.2 Classes of statistical tests 35
5.7.1.3 Tests for normal and binomial data 35
5.7.1.3.1 One-sample tests for normal data 35
5.7.1.3.2 Two-sample tests for normal data 36
5.7.1.3.3 Test for binomial data 37
5.7.1.4 Distribution-free tests for location 38
5.7.1.4.1 Sign tests 38
5.7.1.4.2 Sign rank test 38
5.7.1.4.3 Wilcoxon rank sum test 39
5.7.2 Confidence interval 39
5.7.2.1 Binomial distribution 40
5.7.2.2 Normal (Gaussian) distribution 41
5.7.3 Required sample size for certain confidence levels 42

6 Visualization techniques 43
6.1 Visualization of static data 43
6.1.1 Histograms 43
6.1.2 Barplots 43
6.1.3 QQ-Plots 44
6.1.4 Boxplots 44
6.2 Visualization of dynamic data 45
6.2.1 Line Diagrams 45
6.2.2 Temporal changing Boxplots 45
6.2.3 MMQ-Plots 46

7 Time series modelling 46
7.1 Descriptive characterization 47
7.1.1 Empirical moments 48
7.1.2 Decomposition of time series 49
7.1.3 Determination of the trend component 50
7.1.3.1 Trend function types 51
7.1.3.1.1 Linear trend function 51
7.1.3.1.2 Polynomial trend function 51
7.1.3.1.3 Non-linear trend models 52
7.1.3.2 Trend estimation 52
7.1.3.3 Transformation of time series by filtering 53
7.1.3.3.1 Linear filters 54
7.1.3.3.2 Exponential filters 56
7.1.4 Seasonal component 57

8 Data aggregation 58
8.1 Basic data aggregation operators 58
8.2 Data sources, structures and properties 59
8.2.1 Raw data 59
8.2.1.1 Performance data 59
8.2.1.2 Event data 59
8.2.2 Key Performance Indicators / Parameters 60
8.3 Aggregation hierarchies 60
8.3.1 Temporal aggregation 60
8.3.2 Spatial aggregation 61
8.4 Parameter estimation methods 61
8.4.1 Projection method 61
8.4.2 Substitution method 61

8.4.3 Application of estimation methods 62
8.4.4 Attributes of aggregation operators 62
8.5 Weighted aggregation 63
8.5.1 Perceived QoS 63
8.5.2 Weighted quantiles 64
8.6 Additional data aggregation operators 65
8.6.1 MAWD and BH 65
8.6.2 AVGn 65

9 Assessment of performance indices 65
9.1 Estimation of performance parameters based on active service probing systems 65
9.2 Monitoring concepts 65
9.2.1 Control charts 66
9.2.1.1 Shewhart control charts 66
9.2.1.2 CUSUM and EWMA charts 66
9.2.2 Other alarming rules 66
9.3 Methods for evaluation of objectives 66
9.3.1 Desirability functions 67
9.3.2 Loss functions 67

Annex A (informative): Examples of statistical calculations 68

A.1 Confidence intervals for binomial distribution 68
A.1.1 Step by step computation 68
A.1.2 Computation using statistical software 70
A.1.2.1 Computation in R 70
A.1.2.2 Computation in Excel 71

A.2 Transition from binomial to normal distribution 71

A.3 Definitions of EG 201 769 72

A.4 Calculation of confidence intervals 73
A.4.1 Estimated rate 5 % 73
A.4.2 Estimated rate 50 % 74
A.4.3 Estimated rate 95 % 74
A.4.4 Lower limit of confidence intervals according to Pearson-Clopper formula 75
A.4.5 Upper limit of confidence intervals according to Pearson-Clopper formula 76
A.4.6 Span of confidence intervals according to Pearson-Clopper formula 78

A.5 Different sample sizes 80

A.6 Calculation methods 82
A.6.1 Calculation of quantiles 82

A.7 Reporting of results 84
A.7.1 Methods to use 84
A.7.2 Number of significant decimals 85
A.7.3 Rounding of end results 85

Annex B (informative): Bibliography 86

History 87

Intellectual Property Rights

IPRs essential or potentially essential to the present document may have been declared to ETSI. The information pertaining to these essential IPRs, if any, is publicly available for **ETSI members and non-members**, and can be found in ETSI SR 000 314: *"Intellectual Property Rights (IPRs); Essential, or potentially Essential, IPRs notified to ETSI in respect of ETSI standards"*, which is available from the ETSI Secretariat. Latest updates are available on the ETSI Web server (http://webapp.etsi.org/IPR/home.asp).

Pursuant to the ETSI IPR Policy, no investigation, including IPR searches, has been carried out by ETSI. No guarantee can be given as to the existence of other IPRs not referenced in ETSI SR 000 314 (or the updates on the ETSI Web server) which are, or may be, or may become, essential to the present document.

Foreword

This Technical Specification (TS) has been produced by ETSI Technical Committee Speech Processing, Transmission and Quality Aspects (STQ).

The present document is part 6 of a multi-part deliverable covering the QoS aspects for popular services in GSM and 3G networks, as identified below:

Part 1: "Identification of Quality of Service aspects";

Part 2: "Definition of Quality of Service parameters and their computation";

Part 3: "Typical procedures for Quality of Service measurement equipment";

Part 4: "Requirements for Quality of Service measurement equipment";

Part 5: "Definition of typical measurement profiles";

Part 6: "Post processing and statistical methods";

Part 7: "Sampling methodology".

Part 1 identifies QoS aspects for popular services in GSM and 3G networks. For each service chosen QoS indicators are listed. They are considered to be suitable for the quantitatively characterization of the dominant technical QoS aspects as experienced from the end-customer perspective.

Part 2 defines QoS parameters and their computation for popular services in GSM and 3G networks. The technical QoS indicators, listed in part 1, are the basis for the parameter set chosen. The parameter definition is split into two parts: the abstract definition and the generic description of the measurement method with the respective trigger points. Only measurement methods not dependent on any infrastructure provided are described in the present document. The harmonized definitions given in the present document are considered as the prerequisites for comparison of QoS measurements and measurement results.

Part 3 describes typical procedures used for QoS measurements over GSM, along with settings and parameters for such measurements.

Part 4 defines the minimum requirements of QoS measurement equipment for GSM and 3G networks in the way that the values and trigger-points needed to compute the QoS parameter as defined in part 2 can be measured following the procedures defined in part 3. Test-equipment fulfilling the specified minimum requirements, will allow to perform the proposed measurements in a reliable and reproducible way.

Part 5 specifies test profiles which are required to enable benchmarking of different GSM or 3G networks both within and outside national boundaries. It is necessary to have these profiles so that when a specific set of tests are carried out then customers are comparing "like for like" performance.

Part 6 describes procedures to be used for statistical calculations in the field of QoS measurement of GSM and 3G network using probing systems.

Part 7 describes the field measurement method procedures used for QoS measurements over GSM where the results are obtained applying inferential statistics.

Introduction

All the defined quality of service parameters and their computations are based on field measurements. That indicates that the measurements were made from customers point of view (full end-to-end perspective, taking into account the needs of testing).

It is assumed that the end customer can handle his mobile and the services he wants to use (operability is not evaluated at this time). For the purpose of measurement it is assumed:

- that the service is available and not barred for any reason;
- routing is defined correctly without errors; and
- the target subscriber equipment is ready to answer the call.

Voice quality values measured should only be employed by calls ended successfully for statistical analysis.

However, measured values from calls ended unsuccessfully (e.g. dropped) should be available for additional evaluations and therefore, must be stored.

Further preconditions may apply when reasonable.

1 Scope

The present document describes definitions and procedures to be used for statistical calculations which are related to Quality of Service (QoS) measurements done by serving probing systems in mobile communications networks, especially GSM and 3G networks. Network performance measurements and their related post-processing are only marginally covered in the present document.

2 References

The following documents contain provisions which, through reference in this text, constitute provisions of the present document.

- References are either specific (identified by date of publication and/or edition number or version number) or non-specific.
- For a specific reference, subsequent revisions do not apply.
- For a non-specific reference, the latest version applies.

Referenced documents which are not found to be publicly available in the expected location might be found at http://docbox.etsi.org/Reference.

[1] ETSI EG 201 769: "Speech Processing, Transmission and Quality Aspects (STQ); QoS parameter definitions and measurements; Parameters for voice telephony service required under the ONP Voice Telephony Directive 98/10/EC".

3 Definitions, symbols and abbreviations

3.1 Definitions

For the purposes of the present document, the following terms and definitions apply:

rate: measurement result which is related to the portion of time during which it has been executed

NOTE: The denominator's unit is related to time.

ratio: measurement result which represents a subgroup of all single measurements is related to the total number of executed single measurements

NOTE: Usually, nominator and denominator share the same unit, namely a counter for measurements (subgroup/all).

3.2 Symbols

For the purposes of the present document, the following symbols apply:

$E(x)=\mu$	Expected value of random variable x
$Var(x)=\sigma^2$	Variance of random variable x
σ	Standard deviation of random variable x
$f(x)$	Probability Density Function (PDF) of random variable x
$F(x)$	Cumulative Distribution Function (CDF) of random variable x
$S, x \in S$	Set of discrete values or interval of values the random variable x may take
IR	Set of real numbers
s, s^2	Empirical standard deviation / variance, analogous to σ and σ^2 (theoretical)
q_α	α-Quantile
u_α	α-Quantile of standard normal distribution
$x_{(i)}, x_{(1)}, x_{(n)}$	i-th ordered value, minimum and maximum of a given data set x_i, $i = 1,...,n$

3.3 Abbreviations

For the purposes of the present document, the following abbreviations apply:

3G	Third Generation
ARMA	Auto-Regressive Moving Average
AVGn	Averaging Operator (regarding n days)
BH	Busy Hour
BSC	Base Station Controller
CDF	Cumulative Distribution Function or Cumulative Density Function (used synonymously)
CUSUM	CUmulated SUM
EWMA	Exponentially Weighted Moving Average
GSM	Global System for Mobile communications
KPI	Key Performance Indicator
LSL	Lower Specification Level
MAWD	Monthly Average Working Day
MMQ-Plot	Median-Mean-Quantile Plot
MMS	Multimedia Messaging Service
MOS	Mean Opinion Score
MSC	Mobile Switching Centre
NE	Network Element
PDF	Probability Density Function
QoS	Quality of Service
QQ-Plot	Quantile-Quantile Plot
SMS	Short Message Service
USL	Upper Specification Level

4 Important measurement data types in mobile communications

Appropriate data analysis methods should depend on the type of the given data as well as on the scope of the analysis. Therefore before analysis methods are described, different data types are introduced and differences between them are pointed out.

Four general categories of measurement results are expected when QoS measurements are done in mobile communications.

4.1 Data with binary values

Single measurements related to the topics:

- service accessibility, service availability;
- service retainability, service continuity;
- error ratios, error probabilities;

in general show a binary outcome, i.e. only two outcomes are possible. This means the result of a single trial leads to a result which is either valued positive or negative related to the considered objective. The result may be recorded as decision-results Yes / No or True / False or with numerical values 0 = successful and 1 = unsuccessful (i.e. errors occur) or vice versa. Aggregation of trials of both types allows to calculate the according ratios which means the number of positive / negative results is divided by the number of all trials. Usually, the units of nominator and denominator are the same, namely number of trials.

EXAMPLE: If established voice calls are considered to test the service retainability of a voice telephony system, every successfully completed call leads to the positive result "Call completed", every unsuccessfully ended call is noticed as "Dropped call" which represents the negative outcome. After 10 000 established calls, the ratio of dropped calls related to all established calls can be calculated. The result is the call drop probability.

4.2 Data out of time-interval measurements

Measurements related to the time domain occur in the areas:

- duration of a session or call;
- service access delay;
- round trip time and end-to-end delay of a service;
- blocking times, downtimes of a system.

The outcome of such measurements is the time span between two time stamps marking the starting and end point of the time periods of interest. Results are related to the unit "second" or multiples or parts of it. Depending on the measurement tools and the precision needed, arbitrarily small measurement units may be realized.

EXAMPLE: Someone can define the end-to-end delivery time for the MMS service by a measurement which starts when the user at the A party pushes the "Send" button and which stops when the completely received MMS is signalled to the user at the B party.

4.3 Measurement of data throughput

Measurements related to data throughput result in values which describe the ratio of transmitted data volume related to the required portion of time. The outcome of a single measurement is the quotient of both measures. Used units are "bit" or multiples thereof for the data amount and "second" or multiples or parts thereof for the portion of time.

EXAMPLE: If a data amount of 1 Mbit is transmitted within a period of 60 seconds, this results in a mean data rate of approximately 16,66 kbit/s.

4.4 Data concerning quality measures

Examples are given by the quality of data transfer which may be measured by its speed or evaluations of speech quality measured on a scale, respectively.

Measurements related to audio-visual quality can be done objectively by algorithms or subjectively by human listeners. The outcome of audio-visual quality evaluation is related to a scaled value which is called Mean Opinion Score (MOS) for subjective testing. Thereby two types of quality measurement are distinguished subjective and objective measurements. If quantitative measures are identified which are highly correlated to the quality of interest, this will simplify the analysis. However, if this is not possible, some kind of evaluation on a standardized scale by qualified experts is needed. The result may therefore be given either as the measurement result or as a mark on a pre-defined scale.

EXAMPLE: Within a subjective test, people are asked to rate the overall quality of video samples which are presented to them. The allowed scale to rate the quality is defined in the range from 1 (very poor quality) to 5 (brilliant quality).

Table 4.1 summarizes the different kinds of QoS related measurements, typical outcomes and some examples.

Table 4.1: QoS related measurements, typical outcomes and examples

Category	Relevant measurement types	Examples
Binary values	Service accessibility, service availability	Service accessibility telephony, service non-availability SMS
	Service retainability, service continuity	Call completion rate, call drop rate
	Error ratios, error probabilities	Call set-up error rate
Duration values	Duration of a session or call	Mean call duration
	Service access delay	Service access delay WAP
	Round trip time, end-to-end delay	ICMP Ping roundtrip time
	Blocking times, system downtimes	Blocking time telephony, SGSN downtime
Throughput values	Throughput	Mean data rate GPRS
		Peak data rate UMTS
Content quality values	Audio-visual quality	MOS scores out of subjective testing

5 Distributions and moments

5.1 Introduction

The objective of data analyses is to draw conclusions about the state of a process based on a given data set, which may or may not be a sample of the population of interest. If distributions are assumed, these specify the shape of the data mass up to parameters associated with each family of distributions specifying properties like the mean of the data mass. Location or dispersion shifts of the process will in general result in different parameter estimates specifying the distribution. Therefore the information available from the data is compressed into one or few sufficient statistics specifying the underlying distribution.

Many statistical applications and computations rely in some sense on distributional assumptions, which are not always explicitly stated. Results of statistical measures are often only sensible if underlying assumptions are met and therefore only interpretable if users know about these assumptions.

This clause is organized as follows. Firstly, distributions, moments and quantiles are introduced in theory in clauses 5.2 to 5.4. This part of the document is based on the idea of random variables having certain distributions. Random variables do not take single values but describe the underlying probability model of a random process. They are commonly denoted by:

$$X \sim \text{distribution (parameters)}$$

From the distributional assumptions, moments and quantiles of random variables are derived in theory.

Data is often viewed as being realizations of random variables. Therefore, data analysis mainly consists of fitting an appropriate distribution to the data and drawing conclusions based on this assumption. Clause 5.5 briefly summarizes the estimation of moments and quantiles.

Subsequently, a number of important distributions is introduced in clause 5.6, each of which is visualized graphically to give an idea of meaningful applications. Within this clause, testing distributions are also introduced as they are needed in clause 5.7 for the derivation of statistical tests.

5.2 Continuous and discrete distributions

The main difference between the data types described above can be explained in terms of continuous and discrete distributions. Data with binary values follow a discrete distribution, since the probability mass is distributed only over a fixed number of possible values. The same holds for quality measurements with evaluation results on a scale with a limited number of possible values (i.e. marks 1 to 6 or similar).

On the contrary, time-interval measurements as well as quality measurements based on appropriate quantitative variables may take an infinitely large number of possible values. In theory, since the number of possible outcomes equals infinity, the probability that a single value is exactly realized is zero. Probabilities greater than zero are only realized for intervals with positive width. In practice, each measurement tool will only allow a limited precision resulting in discrete measurements with a large number of possible outcomes. Nevertheless, data from measurement systems with reasonable precision are treated as being continuous.

Formal definitions for continuous and discrete distributions are based on probability density functions as will be described in the following.

5.3 Definition of density function and distribution function

5.3.1 Probability Distribution Function (PDF)

Probability Density Functions (PDF) specify the probability mass either for single outcomes (discrete distributions) or for intervals (continuous distributions).

A PDF is defined as a function $f : IR \rightarrow [0, \infty)$ with properties:

i) $f(x) \geq 0$ for all $x \in S$.

ii) $\int_S f(x)dx = 1$ for continuous distributions or $\sum_S f(x) = 1$ for discrete distributions.

In other words, firstly the values of the PDF are always non-negative, meaning that negative probabilities are neither assigned to values nor intervals, and secondly the summation or integration over the PDF always results in 1 (= 100 %), meaning that any data value will always be realized.

EXAMPLE 1: A PDF for binary data may be given by $f(x) = \begin{cases} 0{,}1: & x = 1 \\ 0{,}9: & x = 0 \end{cases}$, which implies that the probability for a faulty trial (x=1) is 10 %, while tests are completed successfully with probability 90 %.

EXAMPLE 2: For time-interval measurements PDFs may take any kind of shape, as an example a normal distribution with mean 10 (seconds) is assumed here. The PDF for this distribution is given by $f(x) = \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}(x-10)^2\right\}$. Other examples for continuous distributions will follow later on.

EXAMPLE 3: If for instance categories for speech quality are defined as 1 = very poor up to 5 = brilliant, a PDF for the resulting data may be given by $f(x) = \begin{cases} 0{,}1 & : \quad x \in \{1,2,3\} \\ 0{,}4 & : \quad x = 4 \\ 0{,}3 & : \quad x = 5 \end{cases}$.

Figure 5.1 summarizes all three assumed example PDFs for the different data types.

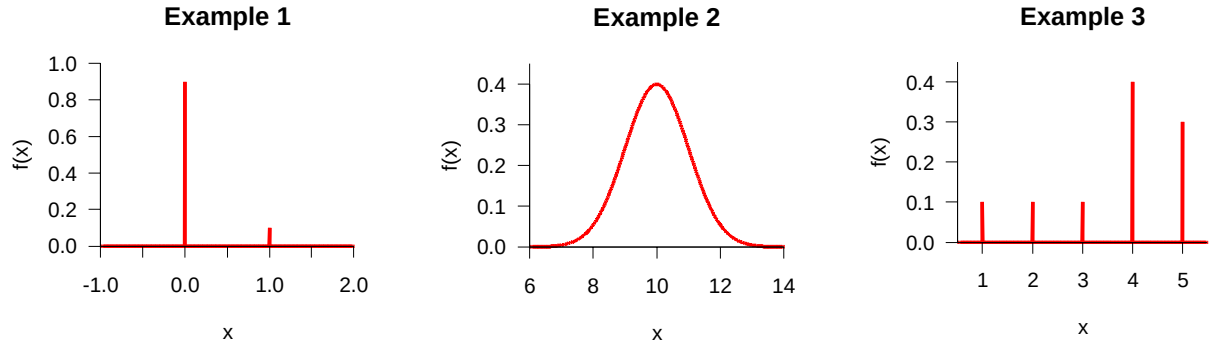


Figure 5.1: Probability Density Functions (PDFs) of examples 1 to 3

5.3.2 Cumulative Distribution Function (CDF)

A Cumulative Distribution (or Density) Function (CDF) is computed from the corresponding PDF as described before by summing (discrete) or integrating (continuous) over the density mass up to the current value.

A function $F : IR \rightarrow [0,1]$ with $F(x) = \sum_{\tilde{x} \leq x} f(\tilde{x})$ for discrete and $F(x) = \int_{-\infty}^{x} f(\tilde{x}) d\tilde{x}$ for continuous distributions is called CDF. This implies $F(x) \rightarrow 1$ for $x \rightarrow \infty$ and $F(x) \rightarrow 0$ for $x \rightarrow -\infty$.

In other words, the value of the CDF corresponds to the proportion of the distribution left of the value of interest. For the three examples from above, the CDFs are given in figure 5.2.

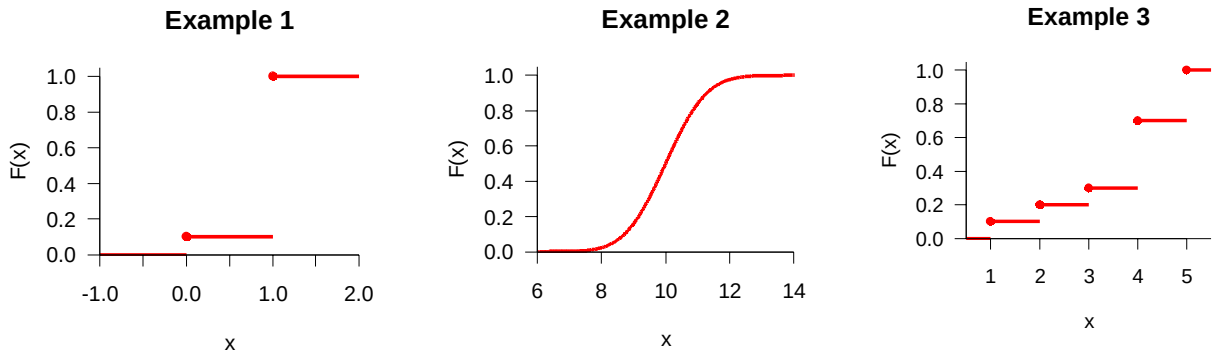


Figure 5.2: Cumulative Distribution Functions (CDFs) of examples 1 to 3

5.4 Moments and quantiles

Moments are main characteristics of distributions. The most important moments are:

- the **expected value** (first moment), specifying the location of the distribution;
- the **variance** (second central moment), specifying the dispersion around the expected value of the distribution; and
- the **skewness** (third central moment), specifying whether a distribution is symmetric or skewed.

These moments are defined as follows.

a) The expected value (first moment, mean) of a random variable *x* with CDF *f(x)* is defined as $E(x) = \int x \cdot f(x)\,dx$ for continuous distributions or $E(x) = \sum x \cdot f(x)$ for discrete distributions, respectively.

b) The variance (second central moment) of a random variable *x* with CDF *f(x)* is defined as $Var(x) = \int (x - E(x))^2 \cdot f(x)\,dx$ for continuous distributions or $Var(x) = \sum (x - E(x))^2 \cdot f(x)$ for discrete distributions, respectively. The square root of the variance called standard deviation, denoted as $\sigma(x)$ is often more informative since it is defined on the original data scale.

c) The skewness (third central moment) of a random variable *x* with CDF *f(x)* is defined as $\int (x - E(x))^3 \cdot f(x)\,dx$ for continuous distributions or $\sum (x - E(x))^3 \cdot f(x)$ for discrete distributions, respectively. A value of zero indicates a symmetric distribution.

EXAMPLE 1: For the CDF from example 1 the moments are given by $E(x) = 0{,}1 \cdot 1 + 0{,}9 \cdot 0 = 0{,}1$, $Var(x) = 0{,}1 \cdot 0{,}9^2 + 0{,}9 \cdot 0{,}1^2 = 0{,}09$ resulting in a standard deviation $\sigma(x) = 0{,}3$. The skewness can be computed as $0{,}1 \cdot 0{,}9^3 + 0.9 \cdot (-0{,}1)^3 = 0{,}072$ indicating that the distribution is not symmetric.

EXAMPLE 2: The moments of the above normal distribution can be computed by partial integration and the fact that the PDF integrates to 1, or by utilizing the properties of normal distributions stating that the mean and standard deviation are the parameters μ and σ of the PDF $f(x \mid \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2\sigma^2}(x-10)^2\right\}$ and that normal distributions are always symmetric. This results in $E(x) = 10$, $Var(x) = \sigma^2 = 1$, which also equals the standard deviation and skewness = 0 for the above example.

EXAMPLE 3: For the CDF of example 3, moments are computed by $E(x) = 0{,}1 \cdot 1 + 0{,}1 \cdot 2 + 0{,}1 \cdot 3 + 0{,}4 \cdot 4 + 0{,}3 \cdot 5 = 3{,}7$, $Var(x) = 1{,}61$ and negative skewness of -1,824.

The moments are computable for all three example PDFs. Nevertheless, they are not always meaningful. In particular in the third example, the possible outcomes are "very poor" to "brilliant", which may be ordered and named 1 to 5 as has been done before, but the expected value of 3,7 does not have a strict meaning. The same applies for higher moments, since the values of the variable of interest are not quantitative, but ordered qualitative.

In case of non-symmetric distributed data, moments may not be appropriate for describing the distribution of interest. An alternative measure of location is given by the **median**, which can be viewed as the point cutting the distribution into halves, namely 50 % of the distribution mass are smaller and 50 % are larger than the median.

More generally, quantiles are defined for each possible percentage. The ***α*-quantile** cuts the distribution in a part of $\alpha \cdot 100$ % of the distribution smaller than this value and $(1\text{-}\alpha) \cdot 100$ % larger than this value. The median as a special case is also called 50 %-quantile.

A **formal definition of quantiles** is for instance given by Mood, Graybill, Boes (1974):

- "The α-quantile q_α with $\alpha \in (0,1]$ is defined as the smallest number q_α satisfying $F(q_\alpha) \leq \alpha$ (for $\alpha = 0$, the minimum value with positive probability or $-\infty$ is defined, respectively)".

Quantiles are easiest illustrated with the examples of CDFs given above, compare figure 5.3. For each CDF, the 5 %, 50 % and 75 %-quantiles are added to the corresponding plot.

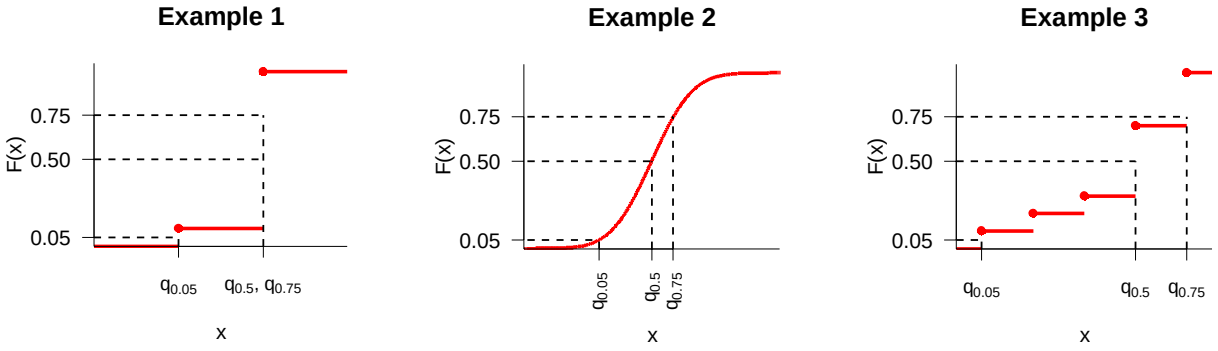


Figure 5.3: Illustration of theoretical quantiles for examples 1 to 3

5.5 Estimation of moments and quantiles

If only samples from the population of interest are available, theoretical moments may not be computed, but have to be estimated empirically.

A sample-based estimator of the expectation of the underlying distribution is given by the **empirical mean** $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$, where $x_i, i = 1, ..., n$ are the sample values. The **variance** of a distribution is commonly estimated by $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$ with resulting **empirical standard deviation** $s = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2}$.

For estimating quantiles, the above definition of theoretical quantiles is commonly replaced by a linear interpolating function. This function on one hand ensures that all quantiles are realized within the range of the empirical distribution (0 %-quantile equals the minimum of the data, 100 %-quantile equals the maximum of the data). The interpolation on the other hand allows a "better guess" of the real quantile if only few data are given and the underlying distribution is continuous. The commonly used computation formula is given by:

$$q_\alpha = (1 - f)x_{(i)} + f \cdot x_{(i+1)}$$

where $i = \lfloor 1 + (n-1)\cdot\alpha \rfloor$, $f = 1 + (n-1)\cdot\alpha - i$ and $x_{(n+1)} := x_{(n)}$.

Here $x_{(i)}$ denotes the *i*-th ordered data value and $\lfloor z \rfloor$ denotes the largest integer less or equal to z, i.e. $\lfloor 3{,}2 \rfloor = 3, \lfloor 4{,}9 \rfloor = 4$. Therefore with the computation of *i*, the quantile is localized depending on the value of α between $x_{(i)}$ and $x_{(i+1)}$. The interpolation between these two values is done according to the deviation *f* between *i* and (1 +(*n*-1)·α).

Examples of empirical CDFs and empirical quantiles for data simulated from the example distributions 1 to 3 are given in figure 5.4. The solid black line represents the empirical quantiles derived by the above formula (from 0 % to 100 %).

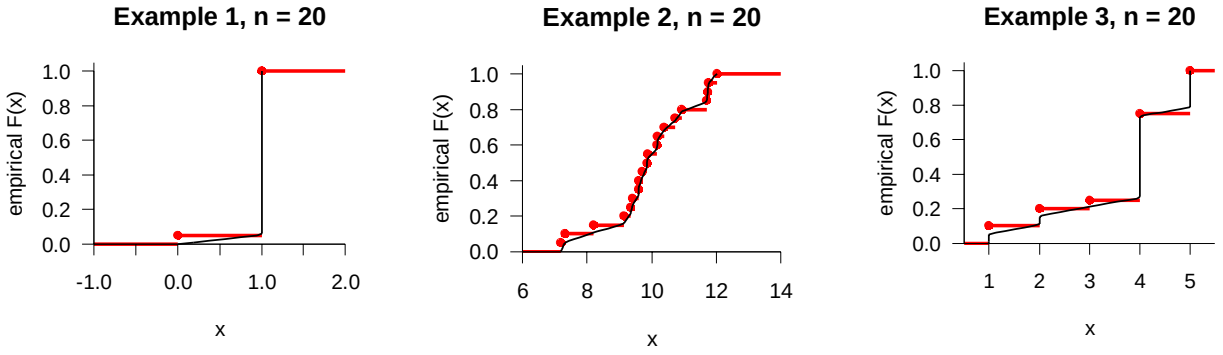


Figure 5.4a: Illustration of empirical CDFs and quantiles for examples 1 to 3

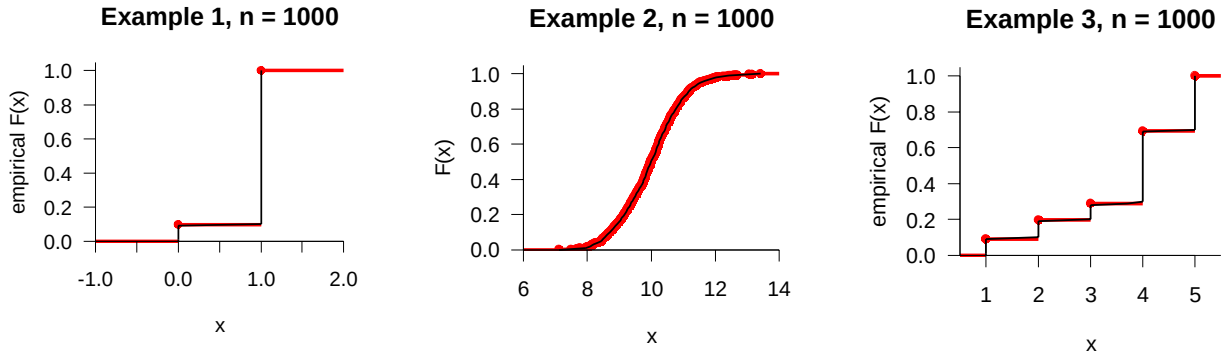


Figure 5.4b: Illustration of empirical CDFs and quantiles for examples 1 to 3

Note that the above estimation procedure should be applied with great care for data sets with only few data values where the underlying distribution is presumably discrete, since the estimated quantiles also take values differing from those contained in the given data set. This can also be seen from figure 5.4a in the plots for samples with sample size n = 20.

5.6 Important distributions

In this clause some of the important distributions related to practical usage in telecommunications are described. Either the mentioned distributions are directly related to measurement results or they are necessary to evaluate these results in a second step. Further relevant distributions may be appended later.

In general, distributions are specified by certain parameters which describe their main characteristics. Commonly, the characteristics are expressed in terms of their moments, i.e. mean value and standard deviation or variance, respectively. Wherever possible, the relevant characteristics are given as well as examples of possible use-cases. In general, continuous and discrete distributions are distinguished further on.

5.6.1 Continuous distributions

A large number of different continuous distributions is available to describe measurement results in a statistical manner. An overview is for instance given by [LAW] or [HART] (see bibliography). For practical purposes in the field of Quality of Service (QoS) probing, the distributions described below are probably the most relevant ones.

5.6.1.1 Normal distribution

The normal distribution, also called Gaussian distribution (or bell-shaped distribution) is used for many natural processes whenever a symmetric continuous distribution seems appropriate. (An example was given before and density functions of further normal distributions are given in figure 5.5.)

	Normal distribution
Notation	$X \sim N(\mu, \sigma^2)$
Parameters	μ, σ
PDF	$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2\sigma^2}(x-\mu)^2\right\}$
CDF	$F(x) = \int_{-\infty}^{x} \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2\sigma^2}(t-\mu)^2\right\} dt$
Expected value	$E(X) = \mu$
Variance	$Var(X) = \sigma^2$
Remarks	Standard normal distribution with $\mu = 0$ and $\sigma = 1$, see clause 5.6.1.1.1

The normal distribution is uniquely specified by its mean and standard deviation. For normally distributed data, about 68 % of the data are realized within the interval [μ - σ, μ + σ], 95 % are realized within [μ - 2σ, μ + 2σ] and 99,7 % are realized within [μ - 3σ, μ + 3σ]. The last interval is also called 6σ-interval which gave the name to the popular "Six-sigma"-courses.

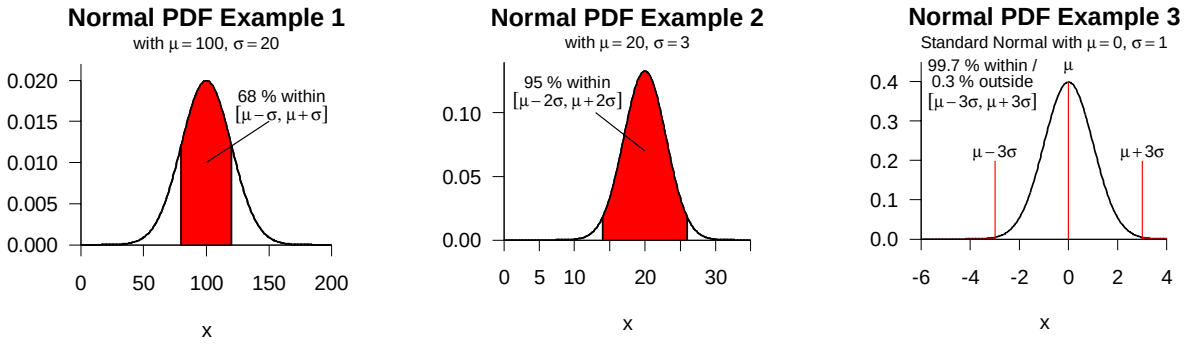


Figure 5.5: Density functions of three different normal distributions

Normally (or nearly normally) distributed data is found quite often in practice, in particular in nature, for example human or animal body heights.

5.6.1.1.1 Standard normal distribution

All normal distributions or normally distributed data can be standardized by subtracting the mean and afterwards dividing by the standard deviation of the distribution or data resulting in a standard normal distribution with mean μ = 0 and standard deviation σ = 1. The inverse computation leads back to the original distribution or data. Therefore, all normal distributions may be reduced to the standard normal, if the parameters μ and σ are known or estimated. Because of this and the fact that many statistical tests are based on the normal distribution, statistical textbooks often provide the quantiles of the standard normal distribution. In particular, the α-quantile of the standard normal distribution is denoted as u_α.

In example 3 of figure 5.5, the density of the standard normal distribution is given.

	Standard normal distribution
Notation	$X \sim N(0,1)$
Parameters	none
PDF	$f(x) = \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}x^2\right\}$
CDF	$F(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} \exp\left\{-\frac{1}{2}t^2\right\} dt$
Expected value	$E(X) = 0$
Variance	$Var(X) = 1$
Remarks	

5.6.1.1.2 Central limit theorem

Another reason for the frequent use of normal distributions (in particular for testing purposes) is given by the central limit theorem, one of the most important theorems in statistical theory. It states that the mean of *n* equally distributed random variables with mean μ and variance σ^2 approaches a normal distribution with mean μ and variance σ^2/n as *n* becomes larger. This holds for arbitrary distributions and commonly the typical shape of the normal distribution is sufficiently reached for $n \geq 4$. For further details about the central limit theorem see [LAW] or [MOOD] (see bibliography).

A number of tools was developed for checking whether data (or means) are normal, namely test procedures like the well-known Kolmogorov-Smirnov goodness-of-fit test (see clause 5.6.6.1.2) among others or graphical tools like histograms or QQ-plots. The mentioned graphical tools will be introduced in clause 6.

5.6.1.1.3 Transformation to normality

As has been seen, the normal distribution is very powerful and can be applied in many situations. Nevertheless, it is not always appropriate, in particular in technical applications, where many parameters of interest have non-symmetric distributions. However, in these situations it may be possible to transform the data to normality. This idea leads for instance to the Log-Normal distribution, which is often assumed for technical parameters.

5.6.1.2 Log-Normal distribution

The distribution of a random variable is said to be Log-Normal, if the logged random variable is normally distributed, which is denoted by $log(x) \sim N(\mu, \sigma^2)$.

	Log-Normal distribution
Notation	$X \sim LN(\mu, \sigma^2)$ or $\log(X) \sim N(\mu, \sigma^2)$
Parameters	μ, σ
PDF	$f_x(x) = \begin{cases} \frac{1}{\sigma x\sqrt{2\pi}} \exp\left\{-\frac{(\ln(x)-\mu)^2}{2\sigma^2}\right\} & \text{if x > 0} \\ 0 & \text{else} \end{cases}$
CDF	$F(x) = \int_{-\infty}^{x} \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{1}{2\sigma^2}(t-\mu)^2\right\} dt$
Expected value	$E(x) = \exp(\mu + \frac{1}{2}\sigma^2)$
Variance	$Var(x) = \exp(2\mu + \sigma^2)(\exp(\sigma^2) - 1)$
Remarks	

Log-Normal distributions are skewed and have heavier upper tails compared to the normal distribution implying a higher variability in the upper quantiles. Density examples for different values of μ and σ are given in figure 5.6 and illustrate that the Log-Normal distribution can take a variety of different shapes.

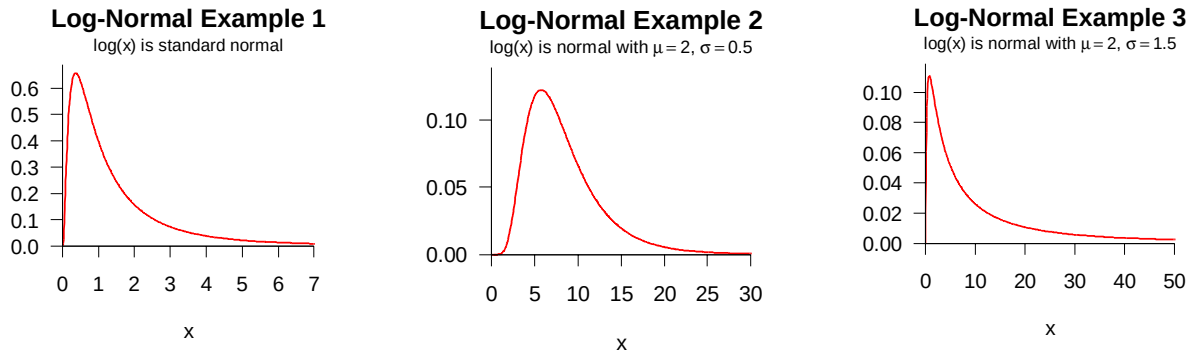


Figure 5.6: Density functions of Log-Normal distributions

5.6.1.2.1 Use-case: transformations

A given data set can be checked whether it is distributed according to a Log-Normal distribution by computing the log of the data values and using one of the graphical tools mentioned before for verifying the normal distribution for the logged data. Empirical mean and standard deviation of the transformed data can then be used for estimating the parameters of the distribution, respectively.

Similarly, other transformation-based distributions can be derived from the normal distribution, for instance for the square-root transformation $\sqrt{x} \sim IN(\mu, \sigma^2)$ or the reciprocal transformation $1/x \sim IN(\mu, \sigma^2)$. A general concept based on power-transformations of x was proposed by Box and Cox (1964).

5.6.1.3 Exponential distribution

For modelling arrival processes, often the negative exponential distribution is used. The relevant parameter for this distribution is λ which symbolizes the life cycle of a process. Concerning arrival processes, λ is named the inter-arrival rate of succeeding events.

	Exponential distribution
Notation	X ~ Exp(λ)
Parameters	$\lambda > 0$
PDF	$f(x) = \lambda \exp(-\lambda x)$ if $x \geq 0$
CDF	$F(x) = 1 - \exp(-\lambda x)$ if $x \geq 0$
Expected value	$E\{X\} = \frac{1}{\lambda}$
Variance	$Var\{X\} = \frac{1}{\lambda^2}$
Remarks	Life-cycle description, survival function: Survival probability $P(X > x) = \exp(-\lambda x)$

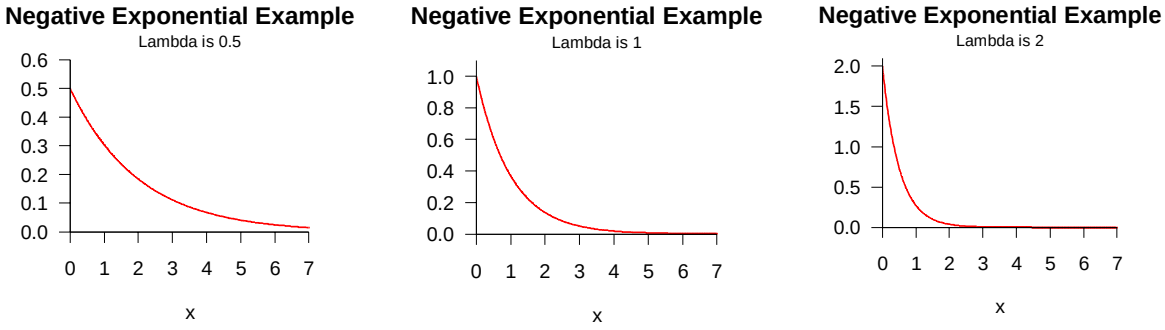


Figure 5.7: Density functions of negative exponential distributions

5.6.1.4 Weibull distribution

The Weibull distribution is a heavy-tailed distribution which means the distribution is skewed with a non-negligible part of the probability mass in the tail. This distribution can be used to describe processes which have a rare frequency, but which are not negligible due to their weight.

	Weibull distribution
Notation	$X \sim Weibull(\alpha, \beta)$
Parameters	α with $\alpha \geq 0$, β with $\beta > 0$
PDF	$f_x(x) = \alpha\beta x^{\beta-1} \exp(-\alpha x^{\beta})$ if $x > 0$
CDF	$F_x(x) = 1 - \exp\left(-\alpha x^{\beta}\right)$ if $x \geq 0$
Expected value	$E\{X\} = \alpha^{-\frac{1}{\beta}} \Gamma\left(\frac{1}{\beta} + 1\right)$ with Γ Gamma function
Variance	$Var\{X\} = \alpha^{-\frac{2}{\beta}} \left[\Gamma\left(\frac{2}{\beta} + 1\right) - \left(\Gamma\left(\frac{1}{\beta} + 1\right) \right)^2 \right]$ with Γ Gamma function
Remarks	Fatigue of material $Weibull(2, \beta)$ is a Rayleigh distribution with parameter β . Rayleigh is used for description of fading effects.

The Gamma function is defined as the integral function $\Gamma(x) = \int_0^{\infty} \exp(-t)\, t^{x-1}\, dt$. One important relationship for the Gamma function is given by $\Gamma(x+1) = x \cdot \Gamma(x)$. For integer values n this relation transforms into $\Gamma(n) = (n-1)!$

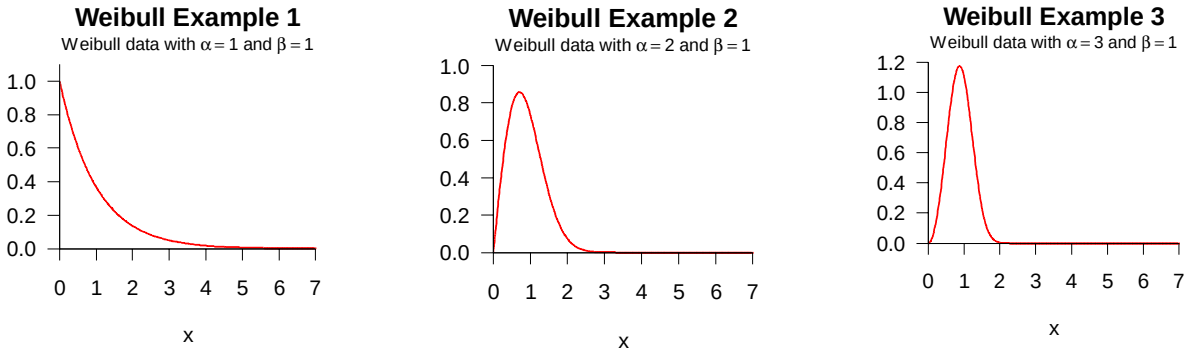


Figure 5.8: Density functions of Weibull distributions

5.6.1.5 Pareto distribution

The Pareto function also models a heavy tailed distribution. One common use-case of this distribution is the modelling of packet-oriented data traffic. For example, the size of HTTP requests and replies as well as FTP downloads can be described as a Pareto function.

	Pareto distribution
Notation	$X \sim Pareto(c, \alpha)$
Parameters	c scale and location parameter α shape parameter
PDF	$f(x) = \alpha \cdot x^{-(\alpha+1)} \cdot c^{\alpha}$ for $x > c$
CDF	$F(x) = 1 - \left(\frac{c}{x}\right)$
Expected value	$E\{X\} = \frac{c}{\alpha - 1}$ for $\alpha > 1$
Variance	$Var\{X\} = \frac{c\alpha}{(\alpha-1)^2 \cdot (\alpha-2)}$ for $\alpha > 2$
Remarks	

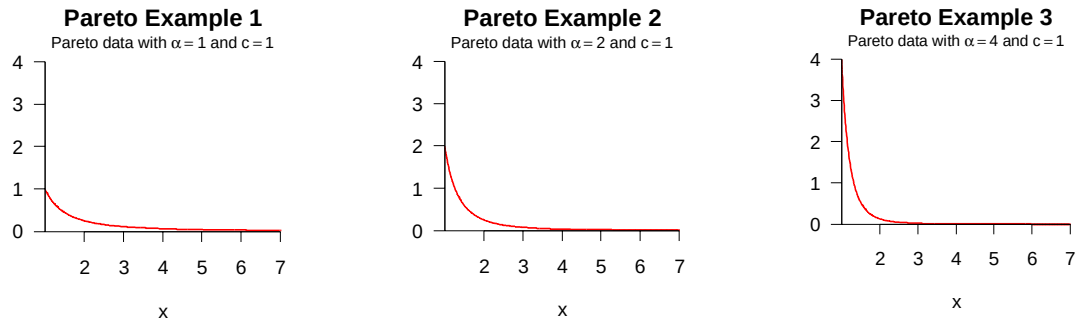


Figure 5.9: Density functions of Pareto distributions

5.6.1.6 Extreme distribution (Fisher-Tippett distribution)

For modelling extremely seldom events with a high and negligible influence, the extreme distribution may be appropriate.

EXAMPLE 1: In service probing, this distribution for example relates to the amount of data which is transferred via FTP data connections. Whereas most of the users generate traffic in the range of some ten or hundred megabytes, at some time single users occur which like to transfer for example 10 gigabytes in one session. When modelling the overall FTP data traffic, these users cannot be neglected due to their immense data volume, but their occurrence probability is very low.

EXAMPLE 2: Concerning insurance cases, single incidents which require a very high financial effort arise when for example an explosion eliminates a complete factory building. Again, due to the high financial effort these cases have be taken into account even they occur rarely.

	Extreme distribution
Notation	$X \sim Extreme(\alpha, \beta)$
Parameters	α shape parameter β scale parameter
PDF	$f(x) = \frac{1}{\beta} \cdot \exp\left(-\frac{x-\alpha}{\beta}\right) \cdot \exp\left[-\exp\left(-\frac{x-\alpha}{\beta}\right)\right]$
CDF	$F(x) = \exp\left[-\exp\left(-\frac{x-\alpha}{\beta}\right)\right]$
Expected value	$E\{x\} = \alpha + \beta\gamma$ with $\gamma \approx 0{,}57721566$ constant of Euler-Mascheroni
Variance	$Var\{x\} = \frac{\Pi^2 \beta^2}{6}$ for $\alpha > 2$
Remarks	

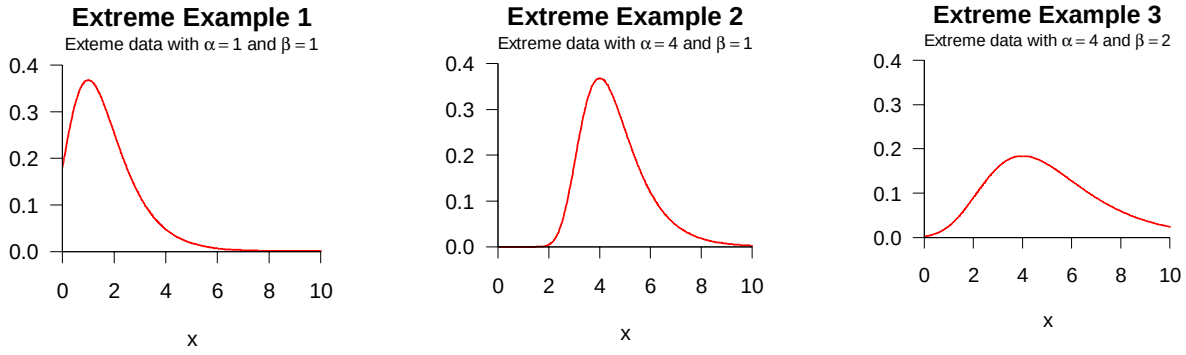


Figure 5.10: Density functions of extreme distributions

5.6.2 Testing distributions

Statistical tests are commonly applied to reject an assumption in favour of an alternative assumption. Therefore, most tests are based on some kind of measure of deviation. This may be the deviation of data from a model assumption or from an assumed mean value, a target value and so on. For computational ease, single deviations are often assumed to be normally distributed.

Based on these concepts, three important testing distributions are introduced in the following, namely the Chi-square-, F- and Student t-distributions.

5.6.2.1 Chi-Square distribution with *n* degrees of freedom

If the results of a service probing is assumed to be the result of a number of independent standard Normal processes, this distribution provides a basis for testing against this assumption. For evaluation purposes concerning the χ^2 distribution, see clause 5.6.4.

A χ^2 distribution represents a combination of n independent random variables Z_1, ..., Z_n where each random variable is standard normal, i.e. $Z_i \sim N(0,1)$. The combination is done according to:

$$\sum_{i=1}^{n} Z_i^2 \sim \chi_n^2$$

The result of this combination is called a "(central) χ^2 distribution with n degrees of freedom".

	(Central) Chi-Square distribution
Notation	$X \sim \chi_n^2$ Random variable $X = \sum_{i=1}^{n} Z_i^2$
Parameters	n : degrees of freedom $Z_1, ..., Z_n$: independent standard normal random variables: $Z_i \sim N(0,1)$
PDF	$f(x) = \frac{1}{2^{\frac{n}{2}} \cdot \Gamma\left(\frac{n}{2}\right)} \cdot x^{\frac{n}{2}-1} \cdot \exp\left(-\frac{x}{2}\right)$ for $x > 0$
CDF	$F(x) = \int_{-\infty}^{x} f(\xi)\ d\xi$ No closed solution available
Expected value	$E\{X\} = n$
Variance	$Var\{X\} = 2n$
Remarks	Combination of n statistically independent $N(0,1)$ random variables (standard normal) Approximation: $F(x) = P(X \le x) \cong \Phi\left(\frac{x-n}{\sqrt{2n}}\right)$

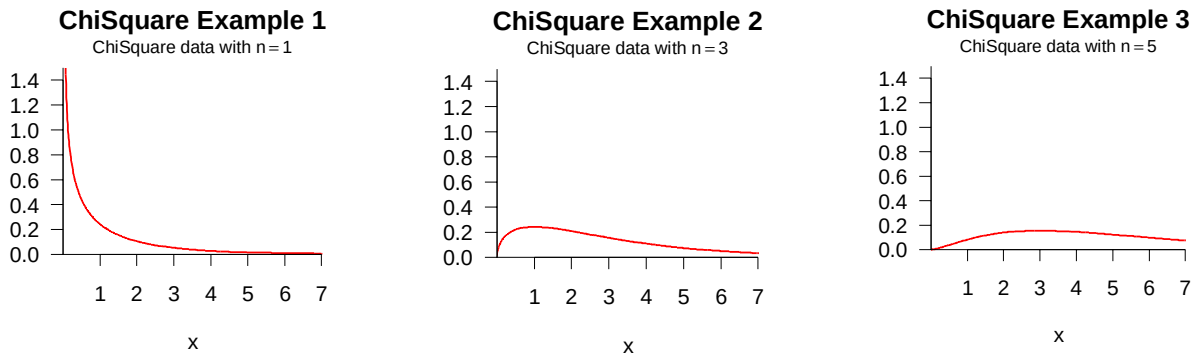


Figure 5.11: Density functions of Chi-Square distributions

5.6.2.1.1 Further relations

The referenced gamma function is defined as the integral function:

$$\Gamma(x) = \int_0^\infty \exp(-t) \cdot t^{x-1} dt$$

Additional useful relations according to this function are:

$$\Gamma(x+1) = x \cdot \Gamma(x)$$

and

$$\Gamma(n) = (n-1)! \text{ if } x = n \text{ is an integer value}$$

5.6.2.1.2 Relation to empirical variance

- If the mean value μ is known, the empirical variance of *n* normally distributed random variables reads $s_\mu^2 = \frac{1}{n} \cdot \sum_{i=1}^{n} (x_i - \mu)^2$. With this piece of information, a chi-square distribution is given for the following expression: $n \cdot \frac{s_\mu^2}{\sigma^2} \sim \chi_n^2$.

- Without knowledge of μ, the empirical variance $s^2 = \frac{1}{n-1} \cdot \sum_{i=1}^{n} (x_i - \bar{x})^2$ estimates the variance of the process. The appropriate relation in this case reads $(n-1) \cdot \frac{s^2}{\sigma^2} \sim \chi_{n-1}^2$.

5.6.2.2 Student t-distribution

If a standard normal and a statistically independent chi-square distribution with n degrees of freedom are combined according to $X = \frac{U}{\sqrt{Z/n}}$, where $Z \sim \chi^2$ (chi-square distributed) and U ~ *N*(0,1) (standard normal distributed), the constructed random variable *X* is said to be ***t*-distributed with *n* degrees of freedom**. Alternatively, the denomination **"Student t-distribution"** can be used.

	Student t-distribution
Notation	$X \sim t_n$ Random variable $X = \frac{U}{\sqrt{Z/n}}$ with $U \sim N(0,1)$, $Z \sim \chi_n^2$, independent.
Parameters	n : degrees of freedom
PDF	$f(x) = \frac{\Gamma\left(\frac{n+1}{2}\right)}{\Gamma\left(\frac{n}{2}\right) \cdot \sqrt{\Pi \cdot n}} \cdot \left(1 + \frac{x^2}{n}\right)^{-\frac{n+1}{2}}$
CDF	$F(x) = \int_{-\infty}^{x} f(\xi)\ d\xi$ No closed solution available
Expected value	$n \geq 2: \ E\{Z\} = 0$
Variance	$n \geq 3: \ Var\{Z\} = \frac{n}{n-2}$
Remarks	The PDF is a symmetric function with symmetry axis $x = 0$. Additional relation for α-quantiles $t_{n;\alpha}$: $t_{n;\alpha} = -t_{n;1-\alpha}$

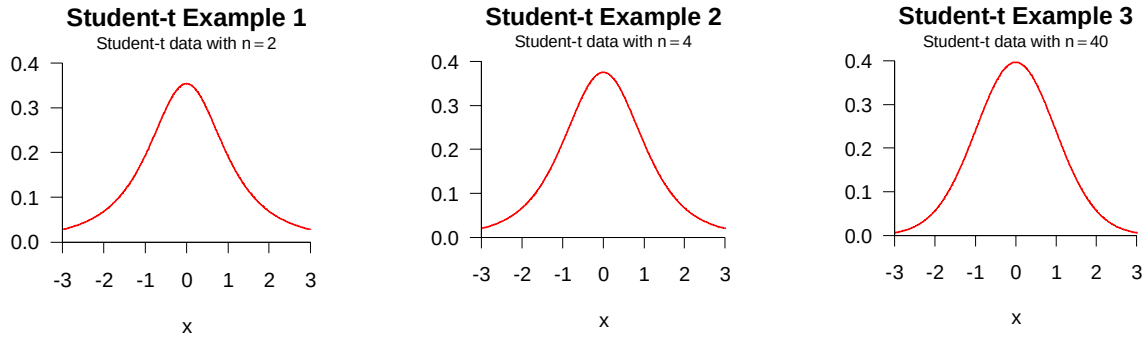


Figure 5.12: Density functions of Student-t distributions

5.6.2.2.1 Relation to normal distribution

It may not be obvious, but *t*-distributions with large number of degrees of freedom may be approximated by a standard normal distribution.

- The standardization of normal variables was covered before: If $X \sim N(\mu, \sigma^2)$, then $(X-\mu)/\sigma \sim N(0, 1)$.
- Consider the case of data assumed to be normal with unknown variance. As stated before, the empirical variance is then related to a chi-square distribution. The empirical mean and variance of n normally distributed ($N(\mu, \sigma^2)$) random variables $X_1, X_2, ..., X_n$ are given by:

$$\bar{X} = \frac{1}{n} \cdot \sum_{i=1}^{n} X_i$$

$$S^2 = \frac{1}{n-1} \cdot \sum_{i=1}^{n} \left(X_i - \bar{X}\right)^2$$

With these relations, the relation between the t-distribution and the *n* normal distributed random variables reads:

$$\sqrt{n} \cdot \frac{\overline{X} - \mu}{\sqrt{S^2}} \sim t_{n-1}.$$

5.6.2.3 F distribution

The F distribution is a combination of m standard normal distributed random variables Y_i and n standard normal distributed random variables V_i which are combined as described below. Again, m and n are called "degrees of freedom" of this distribution.

This distribution is often used for computation and evaluation purposes, for example in relation with confidence intervals for the binomial distribution (Pearson-Clopper formula). In general, it compares two types of deviations, for instance if two different models are fitted.

	F distribution
Notation	$X \sim F_{m,n}$ Random variable $X = \dfrac{\frac{1}{m} \cdot \sum_{i=1}^{m} Y_i^2}{\frac{1}{n} \cdot \sum_{i=1}^{n} V_i^2}$
Parameters	m, n : degrees of freedom $Y_1,....,Y_n$: independent random variables according to $N(0,1)$ $V_1,....,V_n$: independent random variables according to $N(0,1)$
PDF	$f(x) = \dfrac{\left(\frac{m}{n}\right)^{\frac{m}{2}} \cdot x^{\frac{m}{2}-1}}{B\left(\frac{m}{2}, \frac{n}{2}\right)} \cdot \left(1 + \frac{m}{n} \cdot x\right)^{-\frac{m+n}{2}}$ for $x > 0$ with $B(p,q) = \dfrac{\Gamma(p) \cdot \Gamma(q)}{\Gamma(p+q)}$ Eularian beta function
CDF	$F(x) = \int_{-\infty}^{x} f(\xi)\ d\xi$ No closed solution available
Expected value	$n > 2:\ E\{Z\} = \dfrac{n}{n-2}$
Variance	$n > 4:\ Var\{Z\} = \dfrac{2n^2(m+m-2)}{m(n-2)^2(n-4)}$
Remarks	A $F_{m,n}$ related distribution can be interpreted as the quotient of a χ_m^2 distribution and a χ_n^2 distribution multiplied with $\frac{n}{m}$.

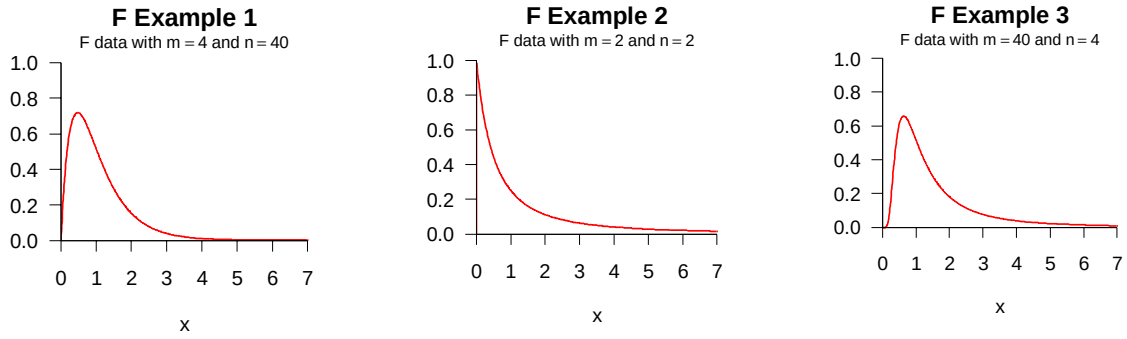


Figure 5.13: Density functions of F distributions

5.6.2.3.1 Quantiles

For quantile computation purposes, the following relations may be useful:

$$F_{n_1,n_2;1-\gamma} = \frac{1}{F_{n_2,n_1;\gamma}}$$

In general, quantile values of this distribution are tabulated.

5.6.2.3.2 Approximation of quantiles

If the desired quantile value cannot be found in tables, the following approximation may be helpful:

If the γ-quantile is wanted with γ in the range $0{,}5 < \gamma < 1$, the relation

$$F_{n_1,n_2;\gamma} \cong \exp(u \cdot a - b)$$

applies where $u = u_\gamma$ is the γ-quantile of the standard normal distribution $N(0,1)$.

The symbols a and b are derived from the following equations:

$$a = \sqrt{2d + cd^2}$$

$$b = 2 \cdot \left(\frac{1}{n_1 - 1} - \frac{1}{n_2 - 1} \right) \cdot \left(c + \frac{5}{6} - \frac{d}{3} \right)$$

$$c = \frac{(u_\gamma)^2 - 3}{6}$$

$$d = \frac{1}{n_1 - 1} + \frac{1}{n_2 - 1}$$

5.6.2.3.3 Relations to other distributions

When the F distribution comes to usage, the following relations may ease the handling of this distribution:

- Relation to t distribution for $n_1 = 1$: $F_{1,n_2;\gamma} = \left(t_{n_2;\frac{1+\gamma}{2}} \right)^2$.
- Relation to χ^2 distribution for $n_2 \to \infty$: $F_{n_1,\infty;\gamma} = \frac{1}{n_1} \cdot \chi_{n_1;\gamma}{}^2$.
- If $n_1 \to \infty$ and $n_2 \to \infty$, the distribution simplifies to: $F_{\infty,\infty;\gamma} = 1$

5.6.3 Discrete distributions

Discrete distributions describe situations where the outcome of measurements is restricted to integer values. For example, the results of service access tests show either that service access is possible (mostly represented by a logical "1" value) or that it is not possible (mostly represented by a logical "0" value). Depending on the circumstances under which such "drawing a ball out of a box" tests are executed, different statistical distributions apply like shown in clauses 5.6.3.1 to 5.6.3.4.

5.6.3.1 Bernoulli distribution

The starting point of different discrete distributions is given by the Bernoulli distribution. It simply describes the probability p of a positive outcome of a single test where only two states are allowed, generally a positive one and a negative one. As soon as more than one single test is executed, further discrete distribution may be applied as shown in the following clauses.

	Bernoulli distribution
Notation	$X \sim Bernoulli(p)$
Parameters	$p \in (0,1)$
PDF	$p(x) = \begin{cases} 1-p & \text{if } x = 0 \\ p & \text{if } x = 1 \\ 0 & \text{otherwise} \end{cases}$
CDF	$F(x) = \begin{cases} 0 & \text{if } x < 0 \\ 1-p & \text{if } 0 \le x < 1 \\ 1 & \text{if } 1 \le x \end{cases}$
Expected value	$E\{X\} = p$
Variance	$Var\{X\} = p \cdot (1-p)$
Remarks	

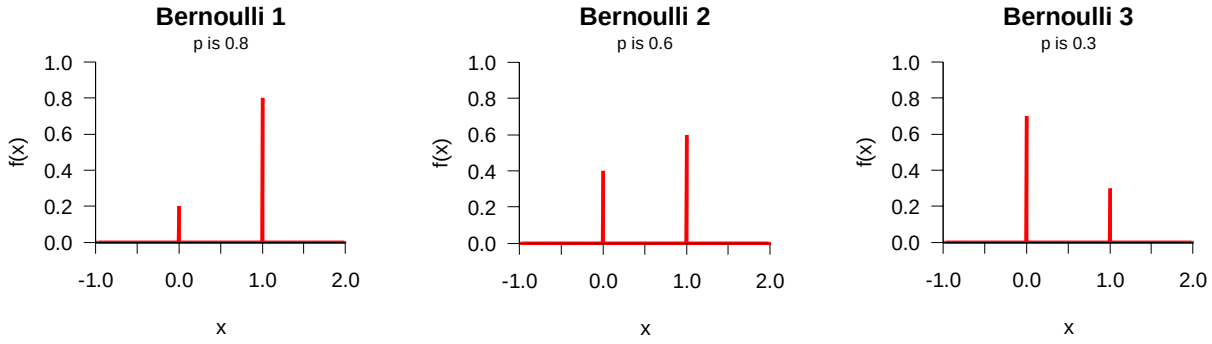


Figure 5.14: Density functions of Bernoulli distributions

5.6.3.2 Binomial distribution

Whenever the outcome of a test is either true or false, the binomial distribution can be applied. In any case where a "black or white" interpretation of results is appropriate, this distribution is able to describe the measurement process. Due to this "yes or no" character, the binomial distribution can be interpreted as the result of different Bernoulli tries. Relevant examples related to service probing are service access issues (e.g. call success rate, SMS send failure ratio, etc.). For a high number of measurement results, the distribution can be replaced by the Normal distribution as a first approximation as shown in clause 5.6.3.3.

Precondition: To determine the CDF of a binomial distribution with relation to different tests, the single events have to be independent from each other. This means that the probability of a successful outcome of different consecutive tests **must not change**. In consequence, this means a memory-less process where the result of a succeeding test is not related to the outcome of its predecessor(s).

	Binomial distribution
Notation	$X \sim Bin(n, p)$
Parameters	n Number of tests m Number of successful test outcomes $p = \frac{m}{n}$ Observed probability of successful outcomes $q = 1 - p$ Observed probability of unsuccessful outcomes
PDF	$P(X = k) = \binom{n}{k} p^k (1-p)^{n-k} = b(n, p, k)$ with $k = 0, 1, 2, ..., n$
CDF	$P(X \le k_0) = \sum_{k=0}^{k_0} \binom{n}{k} p^k (1-p)^{n-k}$ with $k = 0, 1, 2, ..., n$ and $k_0 \ge k$
Expected value	$E\{X\} = n \cdot p$
Variance	$Var\{X\} = n \cdot p \cdot q = n \cdot p \cdot (1-p)$
Remarks	Related to F distribution

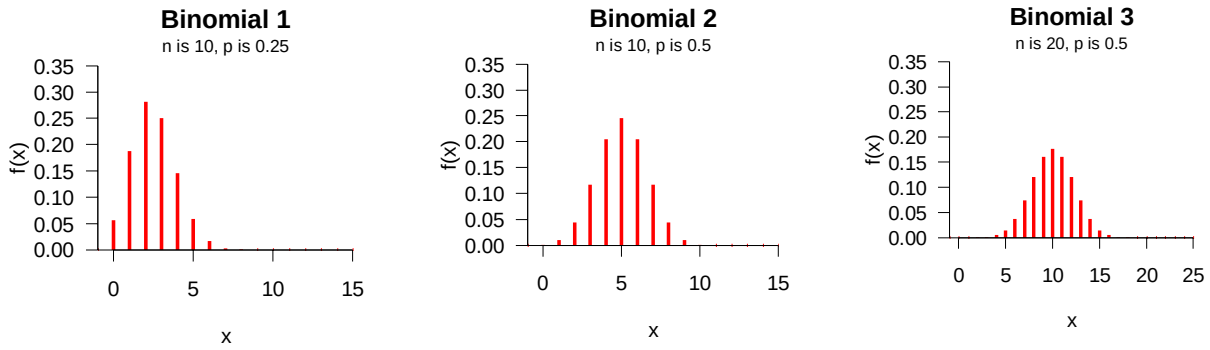


Figure 5.15: Density functions of binomial distributions

For computation purposes, the following relation between the binomial distribution and the F distribution may be useful:

$$P(X < x) = 1 - P\left(F \leq \frac{n-x}{x+1} \cdot \frac{p}{1-p}\right)$$

In this formula, F represents a F distributed random variable with $2 \cdot (x+1)$, $2 \cdot (n-x)$ degrees of freedom.

5.6.3.3 Geometric distribution

The geometric distribution typically describes the following situation: A number of Bernoulli trials is executed consecutively. Each of these trials has a success probability p. By use of the geometrical distribution, one can determine the probability of a successful outcome of a Bernoulli trial after x unsuccessful outcomes.

Scenarios where this computation may be of interest are for example the occurrence of the first success after x failures, or related to service probing, the number of failed service access attempts before the first successful attempt.

	Geometric distribution
Notation	$X \sim G(p)$
Parameters	$p \in (0,1)$
PDF	$p(x) = \begin{cases} p(1-p)^x \text{ if } x \in \{0, 1, ...\} \\ 0 \quad \text{otherwise} \end{cases}$
CDF	$F(x) = \begin{cases} 1-(1-p)^{\lfloor x \rfloor + 1} \text{ if } x \geq 0 \\ 0 \quad \text{otherwise} \end{cases}$
Expected value	$E\{X\} = \frac{1-p}{p}$
Variance	$Var\{X\} = \frac{1-p}{p^2}$
Remarks	

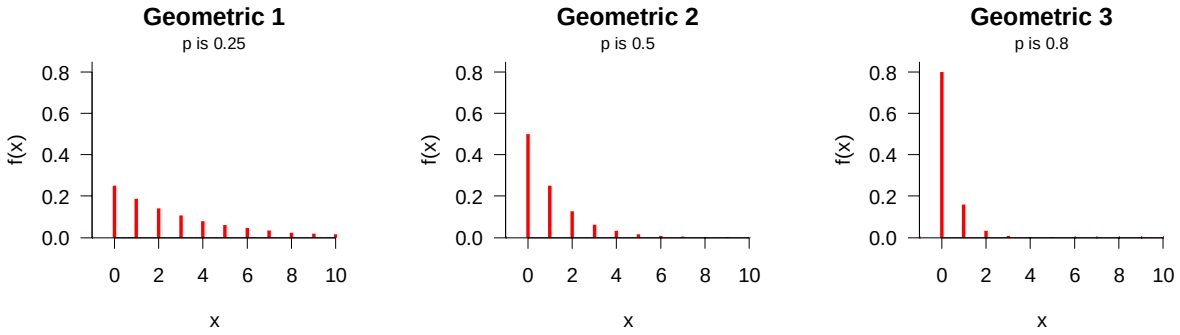


Figure 5.16: Density functions of geometric distributions

5.6.3.4 Poisson distribution

The Poisson distribution is also called "distribution of rare events". Generally, this distribution relates to the number of events within a certain time of period under the precondition that the events occur at a constant rate λ. The Poisson distribution often is used to describe call arrivals in a transmission system, especially the current number of processed service attempts in a system.

	Poisson distribution
Notation	$X \sim Po(\lambda)$
Parameters	λ
PDF	$P(X = k) = \frac{\lambda^k}{k!}\exp(-\lambda)$ with $k = 0, 1, 2, ..., n$
CDF	$P(X \leq k) = \sum_{i=0}^{k} \frac{\lambda^k}{i!}\exp(-\lambda)$ with $k = 0, 1, 2, ..., n$
Expected value	$E\{X\} = \lambda$
Variance	$Var\{X\} = \lambda$
Remarks	Related to χ^2 distribution

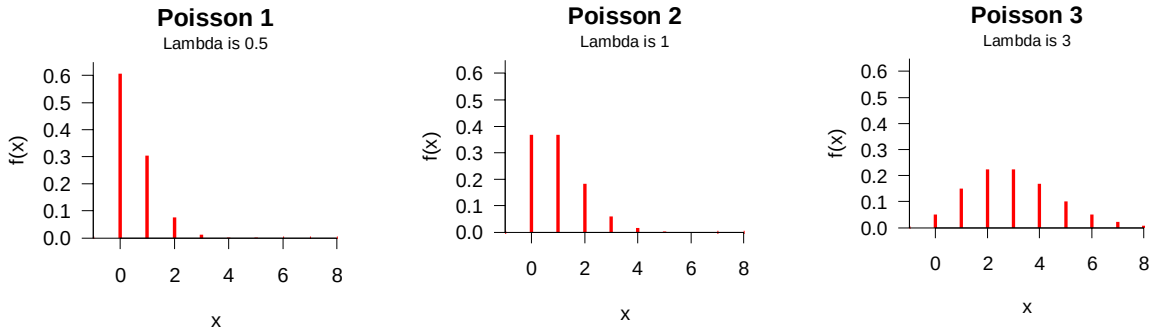


Figure 5.17: Density functions of Poisson distributions

For computation purposes, the following relation between the Poisson distribution and the χ^2 distribution may be useful:

$$P(X \leq x) = 1 - P(\chi^2 \leq 2\lambda)$$

In this formula, χ^2 represents a ${\chi_{2x}}^2$ distributed random variable.

5.6.4 Transitions between distributions and appropriate approximations

Depending on the number of available measurement results, different distributions can be applied to handle the results. In this clause, some useful transitions between common distributions and their required conditions are discussed.

5.6.4.1 From binomial to Poisson distribution

The binomial distribution can be approximated by the Poisson distribution if:

- the probability p is small (rule of thumb: $p < 0{,}1$); and
- the number of executed test cases n is high enough (rule of thumb: $n > 30$).

The approximation of a binomial distributed quantity by a Poisson distribution is given by:

$$P(X = k) \cong \frac{\lambda^k}{k!} \cdot \exp(-\lambda)$$

where the Poisson distribution parameter λ is given by:

$$\lambda = p \cdot n$$

5.6.4.2 From binomial to Normal distribution

If a binomial distribution fulfils the rule of thumb:

$$n \cdot p \cdot q \geq 9$$

then it can be approximated by the Normal distribution:

$$B(n, p) \cong N(n \cdot p, n \cdot p \cdot q)$$

The approximation in detail reads:

$$P(X \leq x) \cong \Phi\left(\frac{X - n \cdot p}{\sqrt{n \cdot p \cdot q}}\right)$$

Especially for smaller numbers of n the following approximation may be more favourable:

$$P(x_1 \leq X \leq x_2) \cong \Phi\left(\frac{x_2 - n \cdot p + 0{,}5}{\sqrt{n \cdot p \cdot q}}\right) - \Phi\left(\frac{x_1 - n \cdot p - 0{,}5}{\sqrt{n \cdot p \cdot q}}\right)$$

5.6.4.3 From Poisson to Normal distribution

According to the Poisson limit theorem, the Poisson distribution can be approximated to the Normal distribution if the distribution parameter λ fulfils the following relation:

$$\lambda = p \cdot n \geq 9$$

which is quite similar to the transition from binomial to Normal distribution.

Then, the approximation reads:

$$P(X \leq k) \cong \Phi\left(\frac{k-\lambda}{\sqrt{\lambda}}\right)$$

5.6.5 Truncated distributions

According to resource constraints of measurement equipment, some measurements have to consider timeout values. By the use of timeouts, the maximal period of time in which measurement results are considered as relevant for the measurement is limited. The resulting density function then is clipped at the right-hand side. Truncation may also occur at both ends of a density function.

For example, if the end-to-end delivery time of some message service is subject of a measurement, the introduction of timeout values may reduce the number of measurement samples. This is because all delivery times which are higher than the defined timeout value are discarded. By discarding some samples, the entirety of data is reduced which means that probabilities describing the measurement may be influenced.

In general, truncation can be described by conditional probabilities. The condition is given by the timeout value. Furthermore, probabilities are then computed under the constraint of the timeout. Truncated Normal and Poisson distributions are covered in more detail by [MOOD] (see bibliography).

5.6.6 Distribution selection and parameter estimation

If a distribution is sought to describe a given data set, two steps have to be carried out. Firstly, an appropriate distribution family (type of distribution) has to be selected and secondly, the corresponding parameters specifying this distribution have to be estimated. Test procedures or graphical methods may be applied for the first step, parameter estimation procedures are needed for the second.

5.6.6.1 Test procedures

The formulation of tests is covered in detail in clause 5.7.1. In this clause, three well-known tests that may be used for checking distributional assumptions are described briefly. It focuses mainly on the fundamental ideas leading to these tests.

All test procedures are based on comparisons between assumed and empirical distribution. That is, from the data on hand, the underlying distribution is guessed and then verified by applying one of the test procedures described in clauses 5.6.6.1.1 to 5.6.6.1.3.

5.6.6.1.1 Chi-Square test

The main idea of the Chi-Square test is to test whether a set of data comes from a normal distribution by building classes and checking whether the expected number of observations and the number of data in each class are similar. If the deviations between both numbers - in terms of squared differences - exceeds a corresponding χ^2-value, the distribution assumed has to be rejected.

5.6.6.1.2 Kolmogorov-Smirnov test

The Kolmogorov-Smirnov test is based on the cumulative distribution functions of the theoretical (assumed) and empirical distribution of the data at hand. The main idea is that the distributional assumption is rejected, if the maximum vertical distance between both Cumulative Density Functions (CDFs) exceeds a critical value.

5.6.6.1.3 Shapiro-Wilk test

Shapiro and Wilk suggested a test procedure that is based on quantiles and related to the QQ-Plot introduced in clause 6. The main idea of this test is to compare the sum of squared deviations between the points of the QQ-Plot and the best fitting straight line with a given χ^2-value.

5.6.6.2 Parameter estimation methods

Most frequently applied methods for parameter estimation are Maximum-Likelihood or Moment estimation methods. For the mean of a normal distribution, both methods yield identical results. For further details compare [MOOD] (see bibliography).

5.7 Evaluation of measurement data

Related to active service probing, certain issues can become much easier to handle if it is able to describe the gathered data in a very compact way. One possible way to reach this aim is to execute different tests and thereby to check some assumptions. These assumptions are stated before any testing is done. They are called "hypotheses".

From a slightly more theoretical point of view, this topic can be expressed as follows:

With every measurement sample some information about the investigated process is retrieved. Since commonly the characteristics of the process are unknown, with every piece of additional information (i.e. every sample) the degree of knowledge increases. This knowledge is formalized by the application of statistical tests or the determination of confidence intervals for the distributional parameters of interest.

The idea of statistical tests and some simple examples are presented in clause 5.7.1. Subsequently, the construction of confidence intervals and the relation between test and confidence intervals will be covered in clause 5.7.2.

5.7.1 Statistical tests

Statistical tests are introduced by specifying the test components first and afterward distinguishing different test classes and giving examples where appropriate.

5.7.1.1 Formulation of statistical tests

Statistical tests are formulated by specifying the following components:

- **(Null-)Hypothesis:** This hypothesis is commonly denoted by H or H_0.

EXAMPLE 1: H: μ = 60.

- **Alternative Hypothesis:** This one is commonly denoted by A or H_1.

EXAMPLE 2: A: $\mu \neq 60$ or A: μ > 60.

- **Test statistic:** A test statistic is derived so that it is sensitive for deviation from the hypothesis in favour of the alternative. That is, the meaning of the test statistic is to notice if in fact A is true instead of H. Test statistics are commonly denoted by T.

- **Testing rule and critical value:** The testing rule states the condition under which H is rejected in favour of the alternative A. So it represents something like a switching condition.

EXAMPLE 3: "Reject H, if $\overline{x} > c$ " or "Reject H, if $|\overline{x}| > c$ ".

The value c is called "critical value".

- **Type I: Error level α :** The probability of rejection H, although true, is controlled by the value α. The specification of α has direct impact on c and thereby on the testing rule. Commonly, the type I error is restricted to 5 % or 1 %, that is α = 0,05 or α = 0,01, respectively.

A statistical test is carried out by specifying all of the above components, computing the test statistic and comparing it with the critical value. Test results are usually documented by reporting the value of the test statistic as well as the corresponding test result. Alternatively, the so called p-value may be reported. This value measures the "significance" or a test result in the way that if the p-value is smaller than the error level α, the hypothesis is rejected, otherwise it may not be rejected. A p-value corresponds to the smallest α-level for which the test would have been rejected.

5.7.1.2 Classes of statistical tests

Commonly, statistical tests are formulated in the way that the alternative makes the statement one wishes to prove in a statistical sense. That is, in general tests seek to reject a given hypothesis and are therefore formulated accordingly. This is done due to the fact that if the Type I error is specified, a rejection implies that the hypothesis is untrue with user-defined certainty.

However, a number of test procedures exist that differ from the mentioned general test philosophy. Some of these examples are the tests used for selecting distributions (compare clause 5.6.6). Their purpose is to support the hypothesis. Nevertheless, strictly speaking it is impossible to proof any equality hypothesis, therefore the test result can either be that there is no hint that the hypothesis is violated or that there is evidence that the assumed distribution is not appropriate. In the following, it is assumed that one wishes to reject the hypothesis in favour of the alternative.

Two major classes of tests are distinguished, namely **one-sample** and **two-sample** tests.

- If a test is based on only one data set for which a reference alternative is to be checked, this is a one-sample test.
- On the other hand, two data sets may be compared by testing for instance the hypothesis H related to the Multimedia Messaging Service (MMS).

  H: MMS-E2E-Delivery Time [this week] > MMS-E2E-Delivery Time [last week]

  against the alternative that the MMS-E2E-Delivery Time was reduced from last week to this week.

Furthermore, tests that are **based on distributional assumptions** and **distribution-free tests** are distinguished. Most distribution-based tests are testing for the location and dispersion / variation of an assumed distribution. For two-sample tests, both samples are assumed to be from the same type of distribution, but possibly with different parameters, for instance different location. In contrast, distribution-free tests should be applied, if there is not enough knowledge about the distribution of the data. However, distribution-free tests are in general less powerful, therefore distribution-based tests should be preferred, if appropriate.

5.7.1.3 Tests for normal and binomial data

In the following clauses, two of the main use-cases of statistical data are taken into concern. These clauses deal with test for normal distributed and binomial distributed data.

5.7.1.3.1 One-sample tests for normal data

If data are from a **normal distribution with known variance** σ_0^2, i.e. $X_1,...,X_n \sim N(\mu,\sigma_0^2)$, three different location tests may be carried out. All of these compare the location of a given sample to a value μ_0, that may be chosen arbitrarily.

- Test for *H:* $\mu = \mu_0$ vs. *A:* $\mu \neq \mu_0$: The corresponding test statistic is given by $T = \left| \sqrt{n} \frac{\overline{x} - \mu}{\sigma_0} \right|$

  H is rejected, if $T > u_{1-\alpha/2}$.

- Test for *H:* $\mu \leq \mu_0$ vs. *A:* $\mu > \mu_0$: The corresponding test statistic is given by $T = \left| \sqrt{n} \frac{\overline{x} - \mu_0}{\sigma_0} \right|$

  H is rejected, if $T > u_{1-\alpha}$ (for $\alpha < 0{,}5$).

- Test for *H:* $\mu \geq \mu_0$ vs. *A:* $\mu < \mu_0$: The corresponding test statistic is again given by $T = \left| \sqrt{n} \frac{\overline{x} - \mu_0}{\sigma_0} \right|$

  H is rejected, if $T < u_\alpha = -u_{1-\alpha}$ (for $\alpha < 0{,}5$).

If data are from a **normal distribution, but the variance is unknown**, i.e. $X_1,...,X_n \sim N(\mu,\sigma^2)$, the variance has to be estimated from the data and the above "normal-tests" are replaced by student *t*-tests. In this case, the variance estimator:

$$s^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \overline{x}\right)^2$$

is applied. Test statistics are replaced as follows: $T = \left|\sqrt{n}\,\frac{\overline{x}-\mu_0}{S}\right|$ or $T = \sqrt{n}\,\frac{\overline{x}-\mu_0}{\sigma_0}$, respectively. Critical values are given by the quantiles of the *t*-distribution: $t_{n-1,\,1-\alpha/2}$, $t_{n-1,\,1-\alpha}$ or $t_{n-1,\,\alpha}$, respectively.

If instead **the variance is unknown** and subject of a test, i.e. $X_1,...,X_n \sim N(\mu,\sigma^2)$ with unknown μ and σ^2, the following tests comparing the variance of a given sample to a value σ_0, that may be carried out.

- Test for *H:* $\sigma = \sigma_0$ vs. *A:* $\sigma \neq \sigma_0$: The corresponding test statistic is given by $T = (n-1)/\sigma_0^2\, s^2$ with $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \overline{x}\right)^2$. *H* is rejected, if $T > \chi^2_{1-\alpha/2,\,n-1}$ or $T < \chi^2_{\alpha/2,\,n-1}$.
- Test for *H:* $\sigma \leq \sigma_0$ vs. *A:* $\sigma > \sigma_0$: The corresponding test statistic is again given by $T = (n-1)/\sigma_0^2\, s^2$ with s^2 as given above. *H* is rejected, if $T > \chi^2_{1-\alpha,\,n-1}$.
- Test for *H:* $\sigma \geq \sigma_0$ vs. *A:* $\sigma < \sigma_0$: Test statistic and empirical variance are as before. In this case, *H* is rejected, if $T < \chi^2_{\alpha,\,n-1}$.

5.7.1.3.2 Two-sample tests for normal data

In the case of two samples, that are to be compared, two very different situations are distinguished. The two samples can either be collected on the same observational units or can be observed independently. If both samples are from the same units, for example measuring the cut-of-call-ratio at different network elements before and after a new piece of software is installed, the two samples are called **paired** and two observations from the same unit will generally be correlated. In this case, the differences between both measurements for each unit are computed and the new observations $D_i = X_i - Y_i$ are assumed to be normal with expectation $\mu_D = \mu_X - \mu_Y$. Then, the above tests for normal data may be applied, for instance to test for $\mu_D = 0$, i.e. $\mu_X = \mu_Y$ to prove that there is a significant difference between both samples.

For independent data from two samples, **both assumed to be normally distributed with the same known variance**, but possibly different expectations, i.e. $X_1,...,X_n \sim N(\mu_X,\sigma^2)$ and $Y_1,...,Y_m \sim N(\mu_Y,\sigma^2)$, tests to compare both means are given as follows.

The following test statistic T is defined for testing the hypotheses:

$$T = \frac{\overline{x}-\overline{y}}{\sigma\sqrt{\frac{1}{n}+\frac{1}{m}}}$$

- Test for *H:* $\mu_X = \mu_Y$ vs. *A:* $\mu_X \neq \mu_Y$: *H* is rejected, if $|T| > u_{1-\alpha/2}$
- Test for *H:* $\mu_X \leq \mu_Y$ vs. *A:* $\mu_X > \mu_Y$: *H* is rejected, if $T > u_{1-\alpha}$
- Test for *H:* $\mu_X \geq \mu_Y$ vs. *A:* $\mu_X < \mu_Y$: *H* is rejected, if $T < u_{\alpha}$

If the **variance is unknown but assumed to be equal for both samples**, the normal distribution is again replaced by a Student t-distribution resulting in the following test procedures.

$$T = \frac{\bar{x} - \bar{y}}{s\sqrt{\frac{1}{n} + \frac{1}{m}}} \text{ with } s = \frac{\sum_{i=1}^{n} x_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)^2 + \sum_{i=1}^{m} y_i^2 - \frac{1}{m}\left(\sum_{i=1}^{m} y_i\right)^2}{n+m-2}$$

- Test for *H:* $\mu_X = \mu_Y$ *vs. A:* $\mu_X \neq \mu_Y$*: H* is rejected, if $|T| > t_{1-\alpha/2,\ n+m-2}$.
- Test for *H:* $\mu_X \leq \mu_Y$ *vs. A:* $\mu_X > \mu_Y$*: H* is rejected, if $T > t_{1-\alpha,\ n+m-2}$.
- Test for *H:* $\mu_X \geq \mu_Y$ *vs. A:* $\mu_X < \mu_Y$*: H* is rejected, if $T < t_{\alpha,\ n+m-2}$.

In general, before carrying out one of the above tests, the **assumption of equal variances has to be verified**. This can be done by using the following test:

- Test for *H:* $\sigma_X^2 = \sigma_Y^2$ vs. *A:* $\sigma_X^2 \neq \sigma_Y^2$: The corresponding test statistic is given by $T = \frac{s_X^2}{s_Y^2}$,

where

$$S_X^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2 \text{ and } S_Y^2 = \frac{1}{m-1}\sum_{i=1}^{m}(y_i - \bar{y})^2$$

5.7.1.3.3 Test for binomial data

For binomial data, tests for the probability of success p may be carried out that compare the sample probability to some specified value p_0. Three one-sample tests may be derived by computing the critical values under the three hypotheses

If m is the number of successful trials, the first hypothesis is rejected, if:

$$m > c_{1-\alpha/2} \text{ or } m < d_{\alpha/2}$$

where

$$c_{1-\alpha/2} = \min_{k \in \{1,\ldots,n\}} \sum_{i=k+1}^{n} \binom{n}{i} p_0^i (1-p_0)^{n-i} \leq \alpha$$

and

$$d_{\alpha/2} = \max_{k \in \{1,\ldots,n\}} \sum_{i=0}^{k-1} \binom{n}{i} p_0^i (1-p_0)^{n-i} \leq \alpha$$

The second hypothesis is rejected if $m > c_{1-\alpha}$ and the third one if $m < d_\alpha$.

An alternative way may be appropriate if large numbers of samples are available (large means $np(1-p) \geq 9$ is fulfilled). In this case, the test statistic:

$$Z = \frac{m - np_0}{\sqrt{np_0(1-p_0)}}$$

can be applied. In this case:

- the first hypothesis is rejected, if |Z| > $u_{1-\alpha/2}$;
- the second one, if |Z| > $u_{1-\alpha}$; and
- the third one, if |Z| < $u_{1-\alpha}$.

5.7.1.4 Distribution-free tests for location

If the location for two sets of random variables shall be compared, but there is not enough knowledge for a distributional assumption, distribution-free test may be applied.

5.7.1.4.1 Sign tests

In the case of paired samples, the differences between both measurements for each unit are again computed as $D_i = X_i - Y_i$. If both distributions have the same location, the probability of $X_i < Y_i$ should equal the probability of $X_i > Y_i$ and both should equal 0,5. Based on this consideration, the following tests may be carried out.

- Test for *H:* $P(X_i > Y_i) = P(X_i < Y_i) = 0{,}5$ vs. *A:* $P(X_i > Y_i) \neq 0{,}5$.
- Test for *H:* $P(X_i > Y_i) \leq 0{,}5$ vs. *A:* $P(X_i > Y_i) > 0{,}5$.
- Test for *H:* $P(X_i > Y_i) \geq 0{,}5$ vs. *H:* $P(X_i > Y_i) < 0{,}5$.

In all cases, the test statistic *T* is given as the number of positive differences D_i. This test statistic is a binomial random variable with $p = P(X_i > Y_i)$. Therefore all of the above stated hypotheses are tested by applying a binomial test as described in some more detail in clause 5.7.1.3.3.

5.7.1.4.2 Sign rank test

For the same situation, another kind of test, namely the sign rank test, may be preferable if the distribution of differences is symmetric around some value δ, that is $P(D_i \leq \delta - a) = P(D_i \geq \delta + a)$ for all real numbers *a*. In comparison to the previous clause, the sign rank test not only uses the signs of the differences between both measurements, but also the absolute values in terms of their ranks.

For each of the following hypotheses, the test statistic $T = \sum_{i=1}^{n} V_i R(|D_i|)$ with $V_i = 1$, if $D_i > 0$ and $V_i = 0$ otherwise and $R(\cdot)$ the rank operator that sorts the entries and gives rank *1* to the smallest entry and rank *n* to the largest, is used as a basis for the test decision.

- Test for *H*: $\delta = 0$ vs. *A*: $\delta \neq 0$.
- Test for *H*: $\delta \leq 0$ vs. *A*: $\delta > 0$.
- Test for *H*: $\delta \geq 0$ vs. *A*: $\delta < 0$.

For the test statistic, we have a distribution with expectation $E(T)=\frac{1}{4}n(n+1)$ and variance $Var(T)=\frac{1}{24}n(n+1)(2n+1)$. The quantiles of the resulting distribution are given in statistical text books on nonparametric methods. However, in case $n \geq 20$, the distribution of $\frac{T-E(T)}{\sqrt{Var(T)}}$ may be approximated by a standard normal distribution.

5.7.1.4.3 Wilcoxon rank sum test

In contradiction to both tests explained before, the rank sum test suggested by Wilcoxon is used for independent samples. It assumes that both samples come from the same kind of distribution, but with a shifted location, that is $X_1,...,X_n$ *and* $Y_1,...,Y_m$ *(*$n \leq m$*)* are independent and have continuous distribution functions F_X and F_Y respectively. These are shifted by δ, i.e. $F_X(x)=F_Y(x+\delta)$. Meaningful hypotheses are the following:

- Test for *H*: $\delta = 0$ vs. *A*: $\delta \neq 0$.
- Test for *H*: $\delta \leq 0$ vs. *A*: $\delta > 0$.
- Test for *H*: $\delta \geq 0$ vs. *A*: $\delta < 0$.

In this situation, for instance a rejection of the third hypothesis, i.e. $\delta < 0$ would imply that the location of the second distribution function is significantly smaller, that is the *y*-values are in general be smaller than the *x*-values.

For the above tests, a sensible test statistic is derived by combining both data sets to one sample and computing ranks by ordering the values according to their size. The test statistic *T* is now given by the sum of all ranks for values from the first sample, i.e. the *x*-values. For this test statistic, expectation and variance are given by:

$$E(T)=\frac{1}{2}n(n+m+1)$$

and

$$Var(T)=\frac{1}{12}n\,m\,(n+m+1)$$

respectively. Again, exact critical values for these tests are not easy to derive, but approximations exist. If $n,m \geq 4$ and $n+m \geq 30$, the distribution of $\frac{T-E(T)}{\sqrt{Var(T)}}$ may be approximated by a standard normal distribution.

5.7.2 Confidence interval

In contrast to point estimators where a single number is used to summarize measurement data (compare methods for estimating moments or quantiles in clause 5.5), confidence intervals describe an interval that covers the true parameter value with a certain probability. Usual probability measures are in the 90 percent range. For example, a confidence interval represents the interval in which the mean of the underlying distribution lies with a probability of 95 percent or with a probability of 99 percent.

Confidence intervals are related to statistical tests in the sense that a confidence interval with a given confidence level, for instance 95 %, contains all Confidence levels are denoted by 1-α, where α corresponds to the Type I error level for tests.

As a rule of thumb the number of samples within a measurement campaign correlate with the reliability of results. In other word: The higher the number of collected samples, the more precise and trustworthy the results are.

The computation of confidence intervals depends heavily on the assumed kind of distribution. In the following, the computation of confidence intervals is described for the binomial and the normal (Gaussian) distribution.

5.7.2.1 Binomial distribution

This clause defines how to compute a confidence interval to the level $1-\alpha$ for p for a binomial distribution.

At first, the computation of a confidence interval $[p_1; p_2]$ according to the binomial distribution depends on the number of tests n which are executed to determine p.

- If the condition $n \cdot p \cdot q \geq 9$ is fulfilled, the binomial distribution can be approximated by the Normal distribution which eases the computation of the according confidence interval.

The values for p_1 and p_2 are then given by:

$$p_1 = \frac{2m + u^2_{1-\frac{\alpha}{2}} - u_{1-\frac{\alpha}{2}} \cdot \sqrt{u^2_{1-\frac{\alpha}{2}} + 4m\left(1-\frac{m}{n}\right)}}{2\left(n + u^2_{1-\frac{\alpha}{2}}\right)}$$

$$p_2 = \frac{2m + u^2_{1-\frac{\alpha}{2}} + u_{1-\frac{\alpha}{2}} \cdot \sqrt{u^2_{1-\frac{\alpha}{2}} + 4m\left(1-\frac{m}{n}\right)}}{2\left(n + u^2_{1-\frac{\alpha}{2}}\right)}$$

with the known parameters m and n from clause 5.6.3.4 (Binomial distribution). The term $u_{1-\frac{\alpha}{2}}$ represents the $1-\frac{\alpha}{2}$ quantile of the standard normal distribution $N(0,1)$. Some examples for different confidence levels α and their according $u_{1-\frac{\alpha}{2}}$ quantile values are given in the following table.

Confidence level $1-\alpha$	α	**Term** $1-\frac{\alpha}{2}$	**Quantile** $u_{1-\frac{\alpha}{2}}$
0,9 ($\hat{=}$ 90 %)	0,1	0,95	1,6449
0,95 ($\hat{=}$ 95 %)	0,05	0,975	1,96
0,96 ($\hat{=}$ 96 %)	0,04	0,98	2,0537
0,97 ($\hat{=}$ 97 %)	0,03	0,985	2,1701
0,98 ($\hat{=}$ 98 %)	0,02	0,99	2,3263
0,99 ($\hat{=}$ 99 %)	0,01	0,995	2,5758
0,999 ($\hat{=}$ 99,9 %)	0,001	0,9995	3,2905

The quantile values can be taken from tabulated quantile values of the standard normal distribution respectively the cumulated distribution function of this distribution.

- If the previous condition is not fulfilled, the confidence interval has to be computed with regard to the binomial distribution itself. In this case the parameters p_1 and p_2 are called **"Pearson-Clopper values"**. In detail, the values p_1 and p_2 represent interval boundaries which fulfil the relations:

$$P(X \geq m) = \sum_{k=m}^{n} \binom{n}{k} p_1^k (1-p_1)^{n-k} = \frac{\alpha}{2}$$

$$P(X \leq m) = \sum_{k=0}^{m} \binom{n}{k} p_2^k (1-p_2)^{n-k} = \frac{\alpha}{2}$$

Using the relation between the binomial distribution and the F distribution with $2 \cdot (x+1)$, $2 \cdot (n-x)$ degrees of freedom (see clause 5.6.2.3):

$$P(X < x) = 1 - P\left(F \le \frac{n-x}{x+1} \cdot \frac{p}{1-p} \right)$$

The Pearson-Clopper values can be determined as:

$$p_1 = \frac{m \cdot F_{2m,2(n-m+1);\frac{\alpha}{2}}}{n-m+1+m \cdot F_{2m,2(n-m+1);\frac{\alpha}{2}}}$$

$$p_2 = \frac{(m+1) \cdot F_{2(m+1),2(n-m);1-\frac{\alpha}{2}}}{n-m+(m+1) \cdot F_{2(m+1),2(n-m);1-\frac{\alpha}{2}}}$$

$F_{n_1,n_2;\gamma}$ represents the γ quantile of a F distribution with degrees of freedom n_1 and n_2 which are tabulated in the literature. An approximation for γ quantiles of the F distribution is given in clause 5.6.2.3.

5.7.2.2 Normal (Gaussian) distribution

Related to the Normal distribution, confidence statements depend on the composition of known and unknown parameters. This means different computations have to applied if mean value and/or variance of the distribution are known. If the parameters are not known, they can be estimated by empirical values (see clause 1.5). Furthermore, confidence statements can be made related to the expected value, to the variance and to the standard deviation.

To sum up, the estimated empirical values of the Normal distribution are:

- Empirical mean $\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$, where $x_i, i = 1,...,n$ are the sample values.
- Empirical variance $s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2$.
- Empirical standard deviation $s = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2}$.

Based on these expressions, the following terms are applied to estimate confidence intervals for the mean value, the variance and the standard deviation of a Normal distribution:

- Confidence interval for mean value μ if variance σ^2 is known:

$$\left[\bar{x} - u_{1-\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}} ; \bar{x} + u_{1-\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}} \right]$$

- Confidence interval for mean value μ if variance σ^2 is unknown:

$$\left[\bar{x} - t_{n-1;1-\frac{\alpha}{2}} \cdot \frac{s}{\sqrt{n}} ; \bar{x} + t_{n-1;1-\frac{\alpha}{2}} \cdot \frac{s}{\sqrt{n}} \right]$$

- Confidence interval for variance σ^2 if mean μ is known:

$$\left[\frac{\sum_{i=1}^{n}(x_i-\varpi)^2}{\chi^2_{n;1-\frac{\alpha}{2}}};\frac{\sum_{i=1}^{n}(x_i-\varpi)^2}{\chi^2_{n;\frac{\alpha}{2}}}\right]$$

- Confidence interval for variance σ^2 if mean μ is unknown:

$$\left[\frac{(n-1)s^2}{\chi^2_{n-1;1-\frac{\alpha}{2}}};\frac{(n-1)s^2}{\chi^2_{n-1;\frac{\alpha}{2}}}\right]$$

- Confidence interval for standard deviation σ if mean μ is unknown:

$$\left[s\cdot\sqrt{\frac{n-1}{\chi^2_{n-1;1-\frac{\alpha}{2}}}};s\cdot\sqrt{\frac{n-1}{\chi^2_{n-1;\frac{\alpha}{2}}}}\right]$$

- Confidence interval for standard deviation σ if mean μ is known:

$$\left[\sqrt{\frac{\sum_{i=1}^{n}(x_i-\varpi)^2}{\chi^2_{n;1-\frac{\alpha}{2}}}};\sqrt{\frac{\sum_{i=1}^{n}(x_i-\varpi)^2}{\chi^2_{n;\frac{\alpha}{2}}}}\right]$$

5.7.3 Required sample size for certain confidence levels

In this clause, the relationship between the number of acquired measurement samples and the resulting confidence level is in the focus.

Whereas in the usual measurement chain at first the samples are collected and afterwards the confidence level is determined, in some situations the reverse procedure may be necessary. For example, during a measurement campaign there may exist preconditions which require a certain confidence level which should be reached during the measurements. The unknown parameter is the number of measurement samples which have to be collected to reach this confidence level and must be determined in advance.

Tables of required sample size depending on desired confidence levels are given in clause A.5. The tables are based on the binomial distribution and the according Pearson-Clopper expressions. Due to this fact they are available and valid for rather small sample sizes like they occur if manual tests are executed.

The tables provide two kinds of information:

- The limits and the range of the confidence interval of the mean for an increasing number of samples whereas the estimated rate is constant.
- The range ("span") of the confidence interval of the mean for a varying estimated rate whereas the number of samples is constant.

Based on this, one can state in advance the maximum span of the confidence interval based on the number of samples which should be gathered.

6 Visualization techniques

In this clause, some useful visualization techniques are presented. This is not meant as a complete overview over all possible methods, but should give some standard techniques and some non-standard alternatives.

In the following a distinction is made between static and dynamic data. By static data, variables are meant which do not change systematically within the time period under consideration, i.e. which are not subject to seasonal or daily influences. Dynamic data on the contrary are data which vary systematically over time. Examples are usage data that show a typical curve with high usage during the day (in particular in the afternoon) and low usage at night.

6.1 Visualization of static data

Visualization techniques for static data assume that the underlying distribution does not change over the considered time period and try do give a compressed overview over this distribution.

6.1.1 Histograms

Histograms compress the information by building classes and counting the number of data values falling into each of the specified classes. The main idea is to represent each class by a bar with area equivalent to the portion of data values included. An example is given in figure 6.1.

Histograms can be viewed as density estimators since the area of the visualized bars adds up to one, smoothed density estimation curved can also be applied as available in most of the common statistical analysis computer packages. The two plots of example 1 in figure 6.1 with different bar width illustrate the concept of histograms. Here one bar in the first plot contains the same number of data values than five successive bars in the second plot, therefore the height of one bar in plot one is given by the mean height of the five corresponding bars in plot two. Histograms even allow bar heights greater than one, if the bar width is small, respectively.

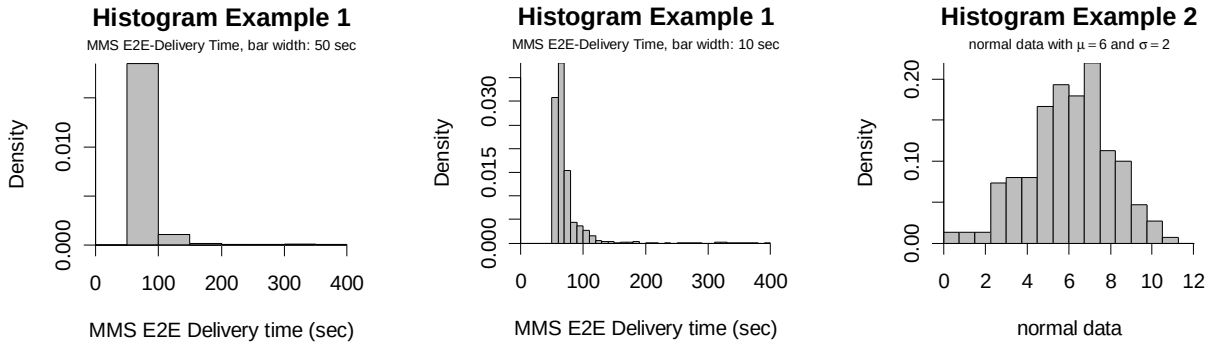


Figure 6.1: Examples of histograms

6.1.2 Barplots

Barplots are suitable for ordinal samples and visualize the total or relative number of elements from a sample with different values of a characteristic of interest. Barplots are used if the distribution of customers to groups with different business state or of trouble tickets with different priorities or other examples of ordinal samples are to be visualized. Since for ordinal samples, the differences between groups are not to be interpreted in a numerical sense, the main difference in comparison to histograms is that the widths of the bars does not have any meaning, only the height corresponds to the total or relative number of elements represented by each bar. Moreover, commonly gaps are left between the bars, to illustrate that ordinal samples are visualized. Examples are given in figure 6.2 where months and priorities are used as example units.

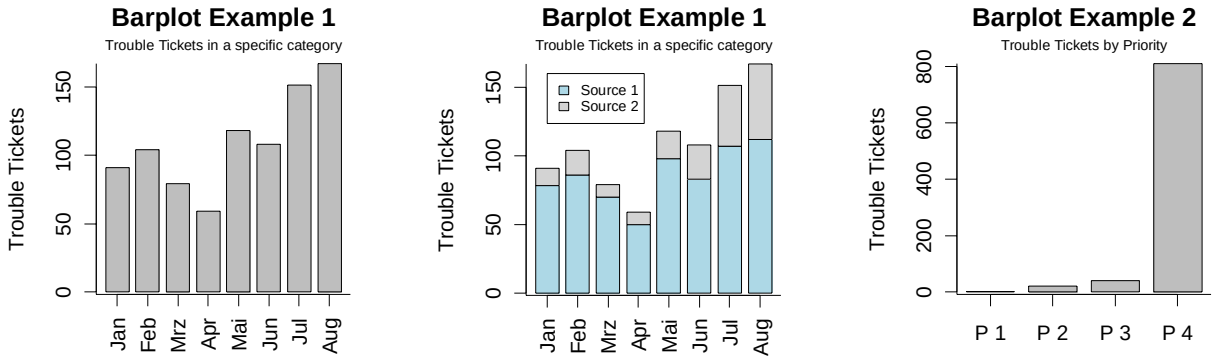


Figure 6.2: Examples of barplots

6.1.3 QQ-Plots

An important tool for checking the normality assumption is the so called Quantile-Quantile-Plot or QQ-Plot. This plot compares the quantiles of two distributions in a scatter plot. In particular the theoretical quantiles of the normal distribution may be compared to empirical quantiles from a sample of interest, but also any other distributional assumptions can be checked, respectively.

In case of a normal QQ-Plot, theoretical quantiles can be taken from the standard normal distribution. The points of the resulting scatter plot should then fall on a straight line with slope corresponding to the empirical standard deviation of the sample. Figure 6.3 gives three example normal QQ-Plots for normal and non-normal samples.

In the first plot, the sample is in fact normal and the normal QQ-Plot also supports the assumption of normal data. For both other plots, non-normal data are simulated to visualize the normal QQ-Plot in cases where the assumptions are violated. In the second example, the entire distribution disagrees with the normal assumption while in example three, only the right tail of the distribution does not agree with the normality assumption.

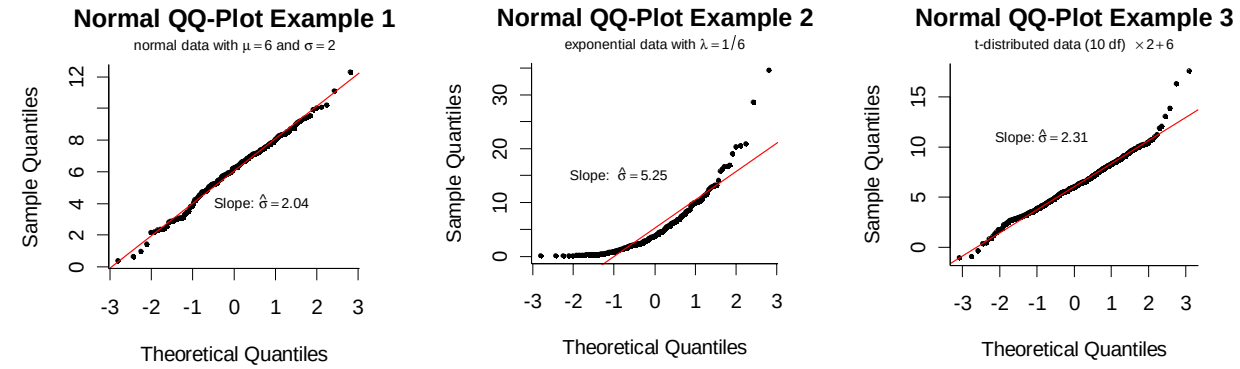


Figure 6.3: Examples of normal QQ-Plots

6.1.4 Boxplots

Boxplots, as the name suggests, consist of a box and some additional shapes called whiskers. These visualize the data information compressed in only a few numbers based on quantiles of the empirical distribution. The end-points of the box are given by the 25 % and 75 %-Quantile (also called Quartiles), the horizontal line is given by the median of the data (50 %-Quantile). Therefore the box contains 50 % of the given data. The whiskers (in the example plots represented by dotted lines) extend to the most extreme data point which is no more than 1,5 times the interquartile range (between the 25 %-Quantile and the 75 %-Quantile) from the box. All data points outside this interval are individually added and may be viewed as outliers. Figure 6.4 gives some boxplot examples.

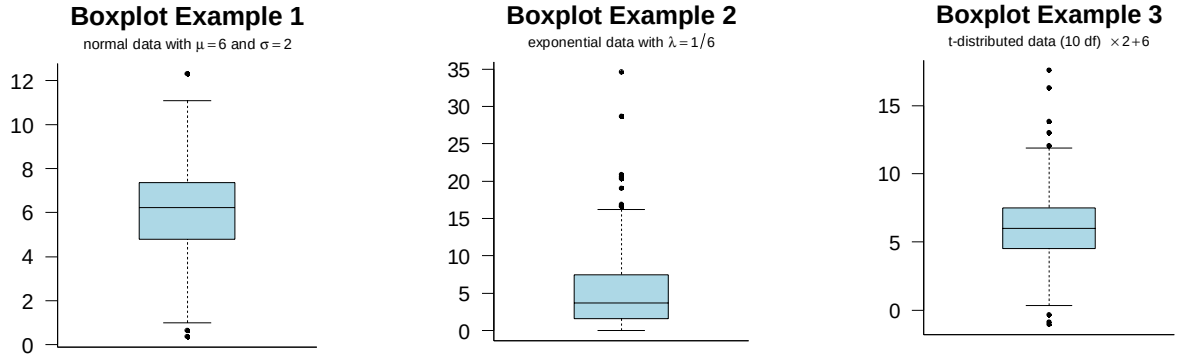


Figure 6.4: Boxplot examples

6.2 Visualization of dynamic data

For dynamic data, visualization techniques should take the dynamic aspect into account. This can be done by visualizing single data points or by using aggregated data values or summary statistics like the mean, respectively. In addition, visualizations as introduced for static data can be compared over time. Boxplots as described in clause 6.1.4 are an adequate tool for characterizing changes over time and will be addressed in clause 6.2.2.

If summary statistics are applied, a chronological classification of the data is needed. This can be done by summarizing a given number of succeeding data points or by summarizing data of a given time period like an hour or a day. In any case, data within a time period or group should be as homogeneous as possible, i.e. changes of the parameter of interest should not be hidden by large classification groups for instance due to long time intervals.

6.2.1 Line Diagrams

Line Diagrams may be based on single data points or values of a summary statistic like the mean. They only provide a visual comparison of the data points over time without any kind of assessment. This can be achieved by adding control limits yielding control charts as will be described in clause 9.2. In figure 6.5, examples of line diagrams are given. If the measurements are not equidistant in time, points of measurement should be specified by points in addition to the connecting line.

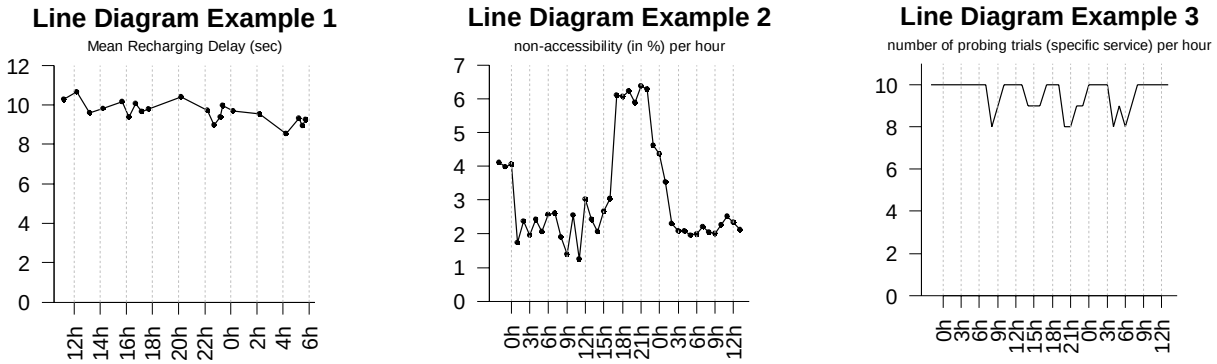


Figure 6.5: Examples of line diagrams

6.2.2 Temporal changing Boxplots

Instead of a single summary statistic, boxplots may be used as a summarizing tool and visualized and compared over time. Boxplots are not only appropriate for comparing empirical distributions over time, but also for unordered groups like the comparison of delay or delivery times for different service providers or vendors. These boxplots are then called parallel boxplots. Examples for both cases are given in figure 6.6.

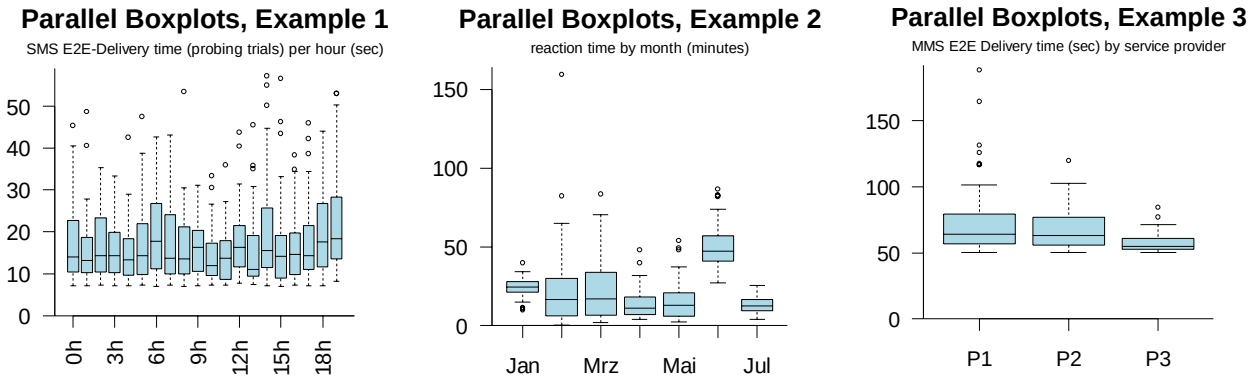


Figure 6.6: Examples of parallel boxplots

6.2.3 MMQ-Plots

Median-Mean-Quantile-Plots (MMQ-Plots) visualize the serial of each of the three statistics median, mean and 95 %-quantile over time in a common plot. The 95 %-quantile characterizes the upper tail of the empirical distribution, while mean and median as measures for location allow conclusions about outliers which will only affect the mean due to its non-robustness. Examples of MMQ-Plots are given in figure 6.7.

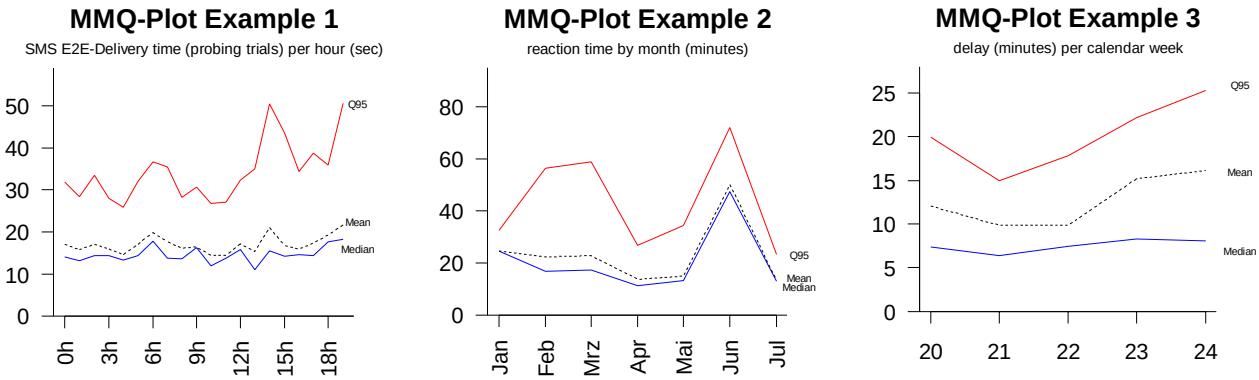


Figure 6.7: Examples of MMQ plots and their temporal behaviour

7 Time series modelling

Beneath stationary processes, on the one hand temporal changes are very interesting. On the other hand there are many cases where an appropriate description of the changes in a system over time have to be handled. Both cases are covered by the so called time series and their appropriate methods.

For example, if measurements in a mobile network are executed for a period of one month with regard to the transmitted traffic volume, a periodic behaviour will be observable. Depending on the hour of day and on the week of day, different traffic situations will be expected.

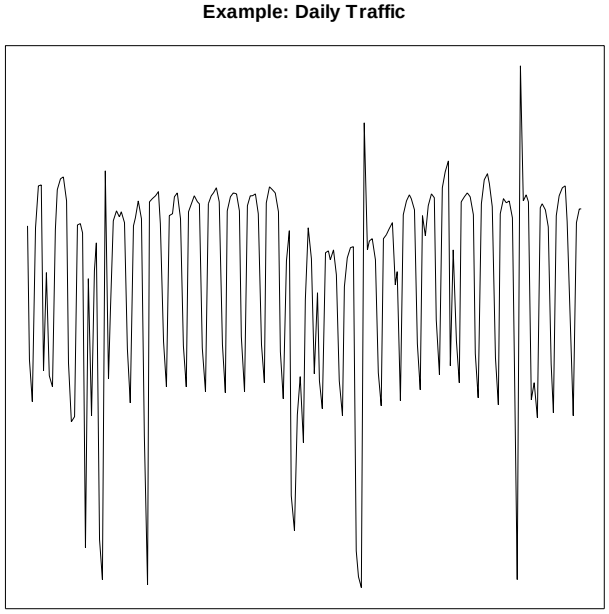


Figure 7.1: Example of daily traffic

Related to time series, four different main areas can be identified:

1) Descriptive characterization

   - This method is based on the principle "let the data speak". Partially, very basic procedures are applied to achieve a description of time series which is as exact and detailed as possible. Especially the method of extracting different components with respect to different time scales is presented.

2) Modelling

   - Time series are interpreted as a realization of a stochastic process which means a sequence of dependent random variables. Under the stationary assumption (i.e. the main characteristics of the process are not varying over time), methods using so called Auto-Regressive Moving Average (ARMA) processes are in the focus.

3) Prognosis

   - If it is assumed that the found stochastical model is valid, it is possible to state the future behaviour.

4) Monitoring

   - Methods in this area are used to model variables which describe technical processes. The aim is to enable the controlling and monitoring of the related processes. Specialized visualization techniques, so called control charts, allow the deployment of these mechanisms in the operational realm. Their main advantage consists in the fact that no further detailed knowledge about statistical methods is required.

7.1 Descriptive characterization

Formally speaking, a time series is an amount of observations x_t which are ordered in ascending order by a time index t. The observations are interpreted as realizations of a random variable X_t. In general, it is assumed that at the point of time the analysis is done, a certain history of observations is available. The history is formally described as a finite amount of parameters N.

$$x_1, x_2, \ldots, x_N$$

Talking in a more practical manner, the observations are represented by certain measurement results which are collected over time and which are analysed in their order of occurrence. Furthermore, the observations can be distinguished according to their timely occurrence: A stochastical process can be observed at fixed points of time which leads to equally spaced intervals between the observations. Another way to observe a process is by executing permanent measurements which deliver measurement results related to some events, so called event data. In effect, the time intervals between consecutive events may vary heavily. In this case it may be appropriate to approximate the situation at fixed points of time. This allows to use mechanisms which are designed for discrete time series.

7.1.1 Empirical moments

Similar to the handling of one-dimensional measurement results, descriptive characteristics (clause 5) can be deployed to describe the main characteristics of a time series. In particular, the arithmetic mean value or the variance respectively the standard deviation are addressed by this issue.

However, these global parameters of time series are only applicable if there is no systematic change in the series, so called stationary time series. In these cases, a movement in a certain direction (i.e. a trend) is not allowed. Concerning non-stationary time series it might be useful to fragment the series in smaller parts. Then, the partial time series can be assumed to be approximately stationary. This allows to use some procedures with a local meaning which are presented in clause 7.1.4.

Beyond this, the question arises if dependencies exist between different observations at different points of time. Corresponding to the covariance, the autocovariance function:

$$c_j = \frac{1}{N}\sum_t (x_t - \bar{x})(x_{t+j} - \bar{x})$$

and the autocorrelation function:

$$r_j = \frac{c_j}{c_0}$$

are defined to measure linear dependencies between succeeding observations of a process. Both functions are defined as functions of the temporal difference (lags) j = - (N-1), .., -1, 0, 1, ..., (N-1) between the observations.

The graphical representation of the autocorrelation function r_j is named **correlogram**. Correlograms are of high relevance for finding cyclic (i.e. periodic) structures in the gathered measurement data.

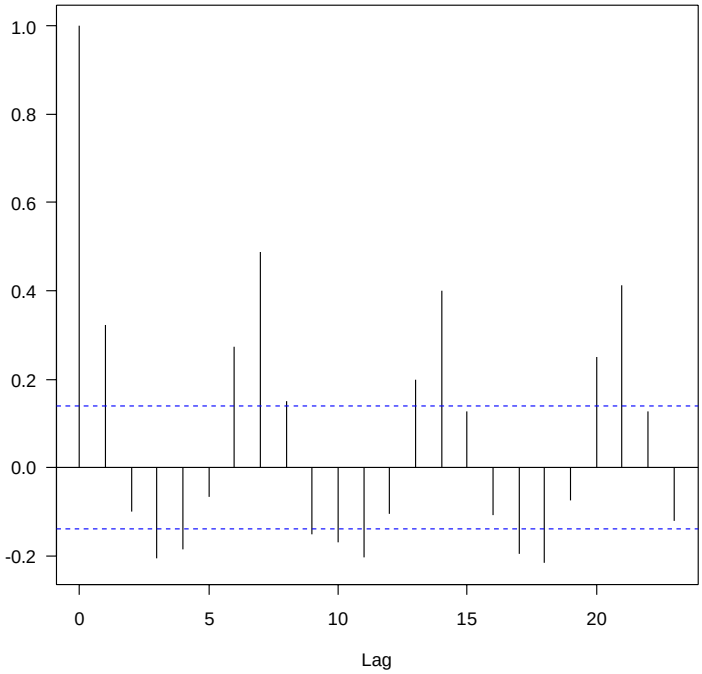


Figure 7.2: Example of correlogram

Furthermore, the autocovariance function again depends on the stationary character of the measurement results because its definition assumes the existence of constant mean values.

7.1.2 Decomposition of time series

The reflection of the example given in the last clause shows the following: Data which is related to the behaviour of customers leads to a mixture of short-term cycles like days and long-term cycles which change on an annual basis. This means that the daily changes are overlaid by for example weekly changes as well as seasonal or yearly modifications.

Now, the aim to reach by the decomposition of time series is the following: The time series should be decomposited to be able to identify the long-term trend of a process. The question which should be answered is: Are there any long-term movements behind the different layered cyclic processes? Especially with respect to management decisions, this information can be of a very high importance.

In general, time series are based on two different assumptions:

- Additive time series:

$$x_t = T_t + K_t + S_t + R_t \text{ for } t = 1, ..., n$$

- Multiplicative time series:

$$x_t = T_t \cdot K_t \cdot S_t \cdot R_t \text{ for } t = 1, ..., n$$

The different parts are in detail:

- T_t (*Trend*) represents the systematic long-term changes of the mean niveau of a time series.
- The economic component K_t includes long-term changes in the model which need not to be regular in any way. The combination of T_t and K_t often is concentrated in terms of a smooth component G_t.
- Cyclical changes are represented by the term S_t. This is the seasonal component of the process.
- R_t stands for an irregular behaviour of the process which is not known in advance. Generally speaking, this component is assumed to be part of a random process which oscillates around the zero level.

If seasonal changes occur with the same amplitude in each period, the additive time series model should be taken into account. The other way round, if seasonal changes change their amplitude with every observation period while they keep their general behaviour, the multiplicative approach may be the better choice.

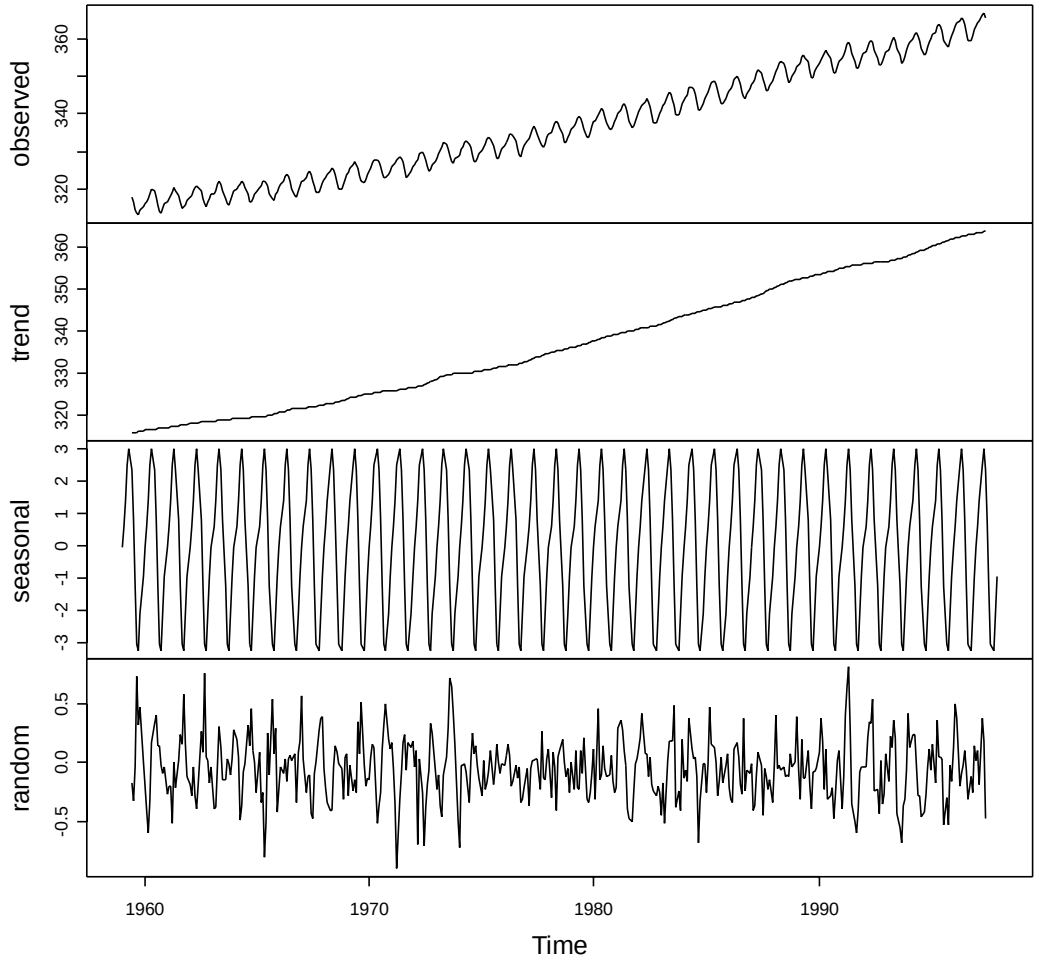


Figure 7.3: Example of decomposition in different components

In general, there is no statement how to process a given time series in an optimal way. Therefore, different approaches or modelling types may lead to different results. Particularly, two different approaches can be distinguished:

- Global component model: The time series is mapped to a global model which is valid for all clauses of the time series and which is adapted to the specific time series. The estimation of the trend component is usually done by the adaption of linear and non-linear regression models based on the method of minimized square values.

- Local component model: In this model, the time series is split up in different clauses. For every clause, a certain local model with partial meaning can be developed. The concatenation of all the local models represents the complete time series. The trend estimation is normally done by filtering mechanisms and non-linear procedures.

Both different models are discussed in the following clauses.

7.1.3 Determination of the trend component

The trend component describes the long-term behaviour of a process. Due to the importance the trend component may have with regard to the management sight, the choice of the appropriate model is one of the main issues right from the start. The use of an incorrect model has a far reaching influence with respect to the quality of the component model.

Furthermore, wrong assumptions may lead to misinterpretations. For example, if a linear trend model is introduced, the conclusions drawn from such a model are restricted to a linear character. It is not possible to describe processes which own a more complex function which such a simple model. If it is done anyhow, the resulting conclusions may be completely wrong.

7.1.3.1 Trend function types

Different types of trend functions are possible. All of them introduce some unknown coefficients a_i which must be determined by calculation or estimation. The subsequent clauses introduce different approaches and their characteristics.

7.1.3.1.1 Linear trend function

The most well-known approach to model a trend function is a linear function. It is assumed that the observations x_t depend on the time index t in a linear manner. This relation can be formalized by the expression:

$$x_t = a_1 + a_2 t$$

It is very easy to interpret this model since the sign of a_1 represents if the time series increases (positive sign) or decreases (negative sign) over time.

7.1.3.1.2 Polynomial trend function

Extending the linear approach, the polynomial approach assumes that a time series can be described as the composition of different functions m:

$$x_t = a_1 m_1(t) + a_2 m_2(t) + \ldots + a_k m_k(t)$$

$m_i(t)$ are arbitrary known functions. It is important that the combination of all the single expressions $a_i m_i(t)$ is linear.

A very simple approach is to define the m_i functions as polynomials of rank (i-1). Then, the approach reads:

$$x_t = a_1 + a_2 t + \ldots + a_k t^{k-1}$$

According to the theory, p + 1 points of a function can be perfectly approximated if a polynomial of rank p is used. This means it is possible to reach a perfect approximation between model and any time series in any case. However, there are two serious disadvantages:

- Resulting models are very complex and cannot be interpreted in a simple manner (compared with the basic trend model).
- The assimilation takes only the available data into account. Therefore, it is not possible to make statements about the behaviour in the future.

Both effects are considered as overfitting effects.

7.1.3.1.3 Non-linear trend models

Lots of different non-linear trend models are available. Because of difficulties to describe all models in a general manner, this clause concentrates on some important cases with a very special meaning:

1) Exponential models:

$$x_t = e^{a_1 m_1(t) + a_2 m_2(t) + \ldots + a_k m_k(t)}$$

2) Power models:

$$x_t = m_1(t)^{a_1} \cdot m_2(t)^{a2} \cdot \ldots m_k(t)^{a_k}$$

Both models can be reduced to linear models if a logarithmic operation is applied. Then, the multiplication respective exponentiation is reduced to a simple addition.

3) Logistic models.

In many use cases, it can be assumed that natural limits exist which can be reached by a time series if observations are done over a longer period of time. For example, the growth of customers in a network follows a function which is shaped like an S. In other words, these processes are constrained by saturation effects.

Formally, these time series can be described the following approach:

$$x_t = \frac{a_1}{a_2 + e^{-a_3 t}}$$

In this case the values of the time series converge to the saturation at the value $G = a_1/a_2$.

7.1.3.2 Trend estimation

The common principle behind the different presented approaches is to determine the unknown parameters a_i. Regularly, this is done by estimating the minimization of a squared expression based on a difference. The difference is built by comparing the measurement value x_t with the according approximation given by the chosen approach. Afterwards, the resulting difference is squared and summed up. For the polynomial approach, the according overall expression reads:

$$Q = \sum_{i=1}^{N} \left(x_t - a_1 m_1(t) + a_2 m_2(t) + \ldots + a_k m_k(t) \right)^2 \rightarrow \text{Minimum}$$

Now, the task is to minimize the expression for Q.

To solve the minimization problem, partial derivatives are calculated. In detail, Q is derived with respect to each of the parameters a_i:

$$\frac{\partial Q}{\partial a_i} \stackrel{!}{=} 0,\ i = 1, \ldots, n$$

A system of so called normal equations is the result of this calculation. Under the assumption of linear independency of the different functions m_i, a closed solution exists so that the different parameters a_i can be identified.

Related to non-linear models, the minimization approach leads to normal equations that cannot be solved explicitly. A further leading approach which might solve these problems is based on linear approximations and is called the Gauss-Newton procedure. Additional information can be found in [BAT] (see bibliography).

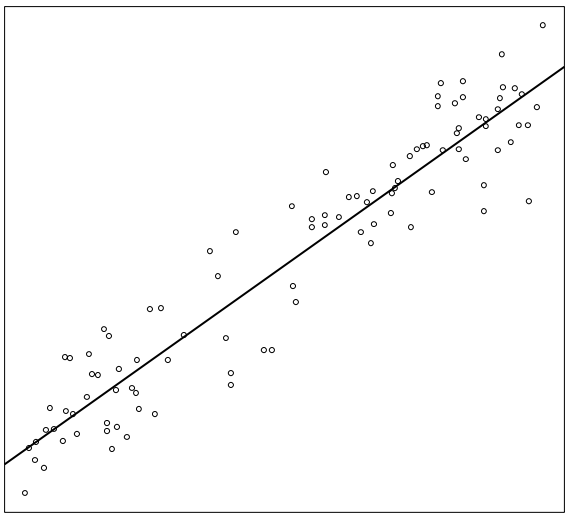


Figure 7.4: Example of a linear model

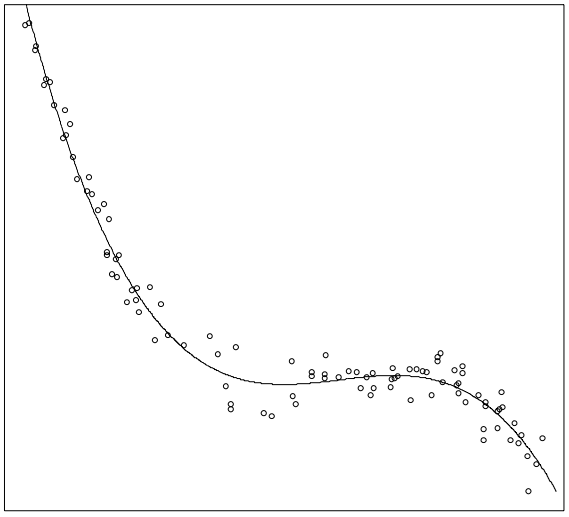


Figure 7.5: Example of a polynomial model

7.1.3.3 Transformation of time series by filtering

Besides the determination of global trends the identification of local trends within the time series is important. The identification of local trends corresponds to the smoothing of a time series by applying filtering functions. One main advantage of this procedure lies in the fact that low-ordered polynomials already lead to acceptable results. This simplification reduces the required computational power.

On the other hand, the main disadvantage of this method is caused by the possibility to get a high number of describing parameters without finding an easy to handle closed solution or model description. In other words: The outcome of this approach may be a smoothened time series, but no model description.

7.1.3.3.1 Linear filters

A very simple approach to reach smoothing effects is the application of a sliding window to a time series. This corresponds to the calculation of a moving average. In general, the according approach can formally described as follows:

A linear filter L is a transformation of a time series x_t into a time series y_t with respect to the relation:

$$y_t = Lx_t = \sum_{i=-q}^{s} a_i x_{t-i} \quad i = s+1, \ldots N-q$$

where (a_{-q}, ..., a_s) symbolize different weights.

The simplest approach is done by "simple moving average". According to the notation given above, it reads:

$$a_i = \frac{1}{2q+1}, i = -q, \ldots q$$

The smoothing effect increases with the number of values taken into account which is the case for increasing values of q.

Reducing the condition for the weighting parameters of the filter to the standardization $\sum a_i = 1$, it is possible to prove that the local approximation based on polynomials is equivalent to the filtering method:

1) The simple moving average is the same as a local trend estimation of the data (x_{i-q},...,x_{i+q}).

2) The filter represented by the polynomial:

$$y_t = \frac{1}{35}\left(-3x_{t-2} + 12x_{t-1} + 17x_t + 12x_{t+1} - 3x_{t+2}\right)$$

represents a local trend estimation according to the squared minimization approach which is based on a polynomial of second order.

The following graphs show some examples for filtering with linear and polynomial filters.

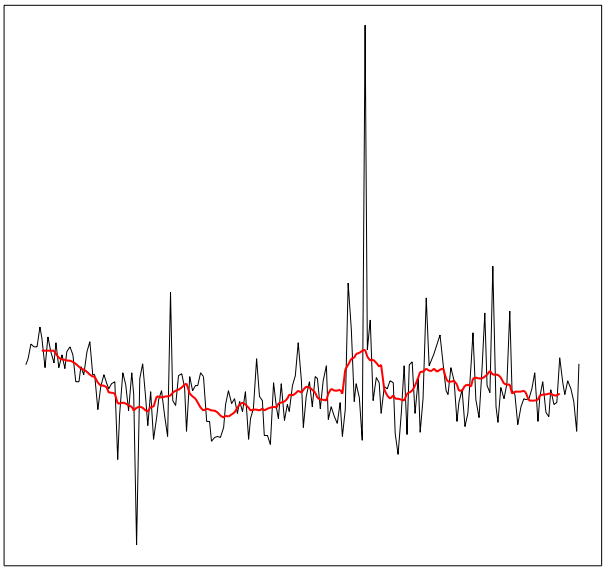


Figure 7.6: Example of linear filter with q = 7

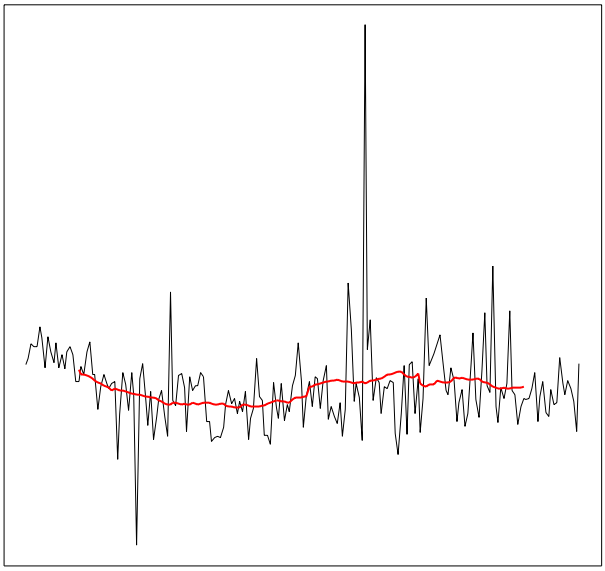


Figure 7.7: Example of linear filter with q = 20

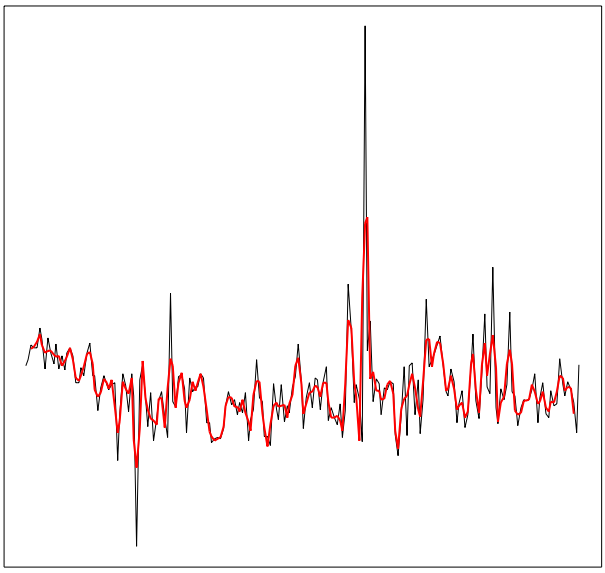


Figure 7.8: Example of polynomial filter

7.1.3.3.2 Exponential filters

Linear filters always use a constant number of weights. Furthermore, a different approach can be interesting which takes into account that older values may be less interesting than newer ones. This is realized by decreasing the weights of older values whereas newer values lead to a higher weighting and is known as an exponential approach. This approach reads in recursive description:

$$y_{t+1} = (1-a)\sum_{i=0}^{\infty} a^i x_{t-i}$$

and is equivalent to the formula:

$$y_{t+1} = ax_t + (1-a)y_t$$

Both expressions are stated in such a way that they can be read as a prediction for the next point of time.

From this equation it can be seen that exponential smoothing also overcomes another limitation of moving averages: older values are weighted with decreasing weights. That is, since a is a number between 0 and 1, the weights [a, $a(1-a)$, $a(1-a)^2$, etc.] show a decreasing magnitude. These are the reasons why exponential smoothing has gained such wide acceptance as a forecasting method.

By rearranging terms in the equation above we can obtain:

$$y_{t+1} = y_t + a(x_t - y_t)$$

In this form the new forecast is simply the old forecast plus a times the error in the old forecast (x_t - y_t). It is evident that when a has a value close to 1, the new forecast will include a substantial adjustment for any error that occurred in the preceding forecast. Conversely, when a is close to 0, the new forecast will not show much adjustment for the error from the previous forecast. Thus the effect of a large and a small a is analogous to the effect of including a small number of observations in computing a moving average versus including a large number of observations in a moving average.

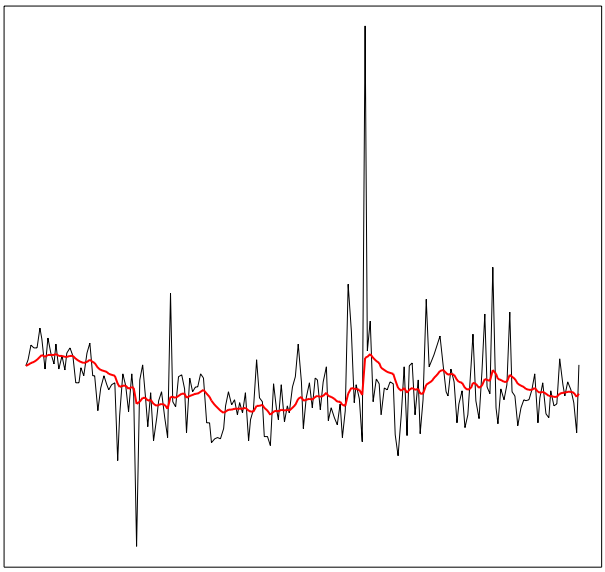


Figure 7.9: Example of exponential filter with a = 0,1

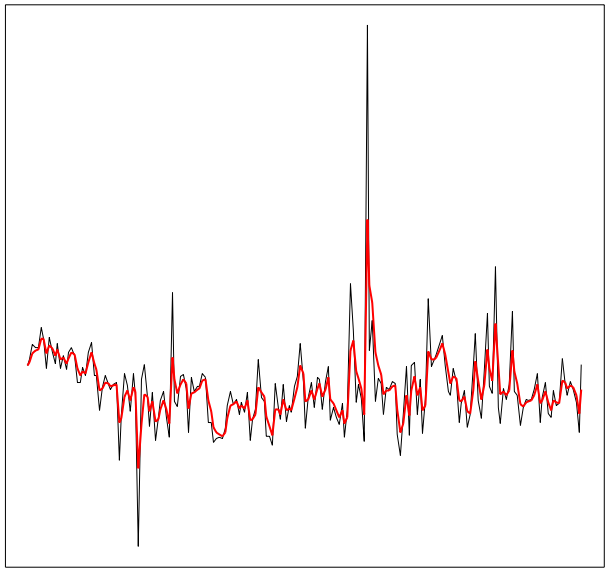


Figure 7.10: Example of exponential filter with a = 0,5

7.1.4 Seasonal component

Especially in data series which are related to the customer's usage, seasonal figures are essentially contained. That means cyclical fluctuations with regular characteristics can be found. Interesting intervals in this area are yearly, monthly and daily periods.

According to two different reasons respective open questions, it may be interesting to eliminate the seasonal influences in the available data. Related to practical issues, the latter procedure is mostly preferred.

- Retrospective view: How would the data have been in the past if no seasonal influences were overlaid?
- Prospective view: What is the long term tendency with respect to the smooth component?

As an example of all the different possible procedures the so called "mean phase" procedure is explained. This procedure is one of the easiest of the available procedures. It is suitable to achieve the elimination of a fixed seasonal component of a time series without a trend component. This means within the data of the time series no trend component is allowed. It must have been removed before by a trend elimination mechanism.

The procedure can be subdivided into four different steps. Generally, it is assumed that the time series x_1, ..., x_N can be separated in different parts, the so called phases *p*, each with a length of *n* data elements. Formally, this relation is given by:

$$(x_t), t = 1, \ldots N \rightarrow (x_{i,j}), i = 1, \ldots p,\ j = 1, \ldots, n$$

The first index *i* describes the number of the segment or the phase whereas the second index *j* represents the consecutive number within this phase. For example, if a data series contains data of a period of 5 years on a monthly basis, it can be described by the parameters $p = 5$ (representing 5 phases, each for one year) and $n = 12$ (representing the 12 months within each year).

The following calculations implement the already mentioned four steps to achieve data without underlying seasonal impacts:

1) Calculation of the average of different phases:

$$\bar{x}_j = \frac{1}{p}\sum_{i=1}^{p} x_{i,j}$$

2) Calculation of the total average:

$$\bar{x} = \sum_{j=1}^{n} \bar{x}_j$$

3) Calculation of seasonal indices (seasonal factors):

$$s_j = \frac{\bar{x}_j}{\bar{x}}$$

Related to the last mentioned calculation, the averaged phases $\bar{x}_j$ are set into relation to the total average $\bar{x}$. In the example, the average of January is for example divided by the total average. This step is done for all of the different monthly averages. If January data is much above average, then $s_i > 1$. According to this, if January data is much below average, $s_i < 1$ will be the result.

4) Calculation of seasonally adjusted values:

$$y_{i,j} = \frac{x_{i,j}}{s_j}, i = 1,\ldots p,\ j = 1,\ldots n$$

This step concludes the basic calculation scheme related to the determination of the seasonal component.

8 Data aggregation

Depending on the objective, i.e. for monitoring or reporting purposes, different types of aggregation may be of interest. Firstly, different granularities in time and space may be needed. Secondly, weighting of data may be considered for instance to generate the QoS perceived by the customer. This may be more or less straightforward, if the smallest time granularity is evenly spaced and the full information at any time point or interval is available, i.e. there are no missing data. However, data aggregation becomes more challenging if event data are considered, like data from active probing systems, or if data are missing and substitution algorithms are needed for defining meaningful results at higher aggregation levels.

In the following, after presenting some basic aggregation operators, different data sources and corresponding structures are distinguished, temporal and spatial aggregation levels are defined and estimation methods for incomplete data are suggested. Based on a summary of desirable attributes of aggregation procedures, an aggregation strategy is suggested and discussed. Subsequently, weighted aggregations are motivated and weighting methods are introduced.

8.1 Basic data aggregation operators

Most common data aggregation operators are **sums** and **means**. Sums are applied if the total number of events, usage, etc. within a given time period is of interest, while means are commonly applied if some kind of averaging is needed. However, in particular for data out of time-interval measurements, means may not lead to interpretable and convincing aggregation results, therefore other summarizing statistics like the minimum, maximum, median and other quantiles of data are also used as aggregation operators. For quality of services measures like accessibilities, retainabilities and so on, ratios in the sense of fractions are used.

The combination of aggregation operators in a row might raise problems, even if the same operator is used at each single step. If the data basis is complete, the combination of sums on different aggregation levels, that is summing sums of lower levels for a higher level aggregation result, ensures a meaningful interpretation. If the considered time intervals are equidistant in addition, the same holds true for the combination of means on different aggregation levels. Minimum and maximum operators are also examples where this kind of combination is possible and meaningful. However, for other quantiles like the median or Q95, it is not recommended to base aggregations on higher levels on the results of lower aggregation levels, since for instance the median of subgroup-medians is not guaranteed to be near the overall median. Aggregation methods for fractions will be discussed later.

If different aggregation operators are combined one after another, the resulting values should be interpreted with great care. For instance the minimum and maximum of mean values from lower aggregation levels should not be mistaken as the range of the original data values. However, one can think of many examples where this kind of combination yields meaningful results that are of interest, for instance if different BSCs are compared with regard to QoS parameters, the median or quantiles of the values for all BSCs may be used as cut-points to identify BSCs performing particularly good or bad.

8.2 Data sources, structures and properties

In the following, a distinction between raw data that may result from data sources with different attributes and parameters that are defined based on these raw data is made.

8.2.1 Raw data

For measuring QoS, data from a number of different data sources are used. These "raw data" are based on different measurement systems with possibly different granularities, differences due to release changes of the underlying systems and so on. Therefore raw data often come with a number of attributes that need to be taken into account for aggregation. Here performance data and event data are considered, although other data sources and types could also provide valuable information about QoS (like fault management data).

In the situation that not all data are available - which is a common problem not only in mobile communications - raw data are rarely substituted or adjusted, but are stored with the full source information to allow suitable parameter definition and estimation. This is often only possible by applying all available reference knowledge, for instance which or how many cells were supposed to deliver data for a given time period.

8.2.1.1 Performance data

Most performance data are given by Network Element (NE) counters. Due to different underlying systems or releases, these may be available in different temporal granularities, like values for 15 minutes from one system and values for hours from another system, respectively. Here basic operations are needed to enable a common data format in order to ensure that values are comparable and basic aggregations are needed for total NE counter values independent of underlying systems. In addition to results of basic aggregations, the total number of aggregated values or even additional detailed reference information needs to be stored.

Examples for performance data are the number of calls per hour per cell, the total number of call minutes per quarter-hour per cell or the number of started WAP-sessions per hour per cell.

8.2.1.2 Event data

Billing data are one example of event data that may provide information about the QoS. On the other hand, results from active probing systems are also in general not equally spaced over time. This may be due to varying duration of trials, possibly depending on network performance or other influences. Also there may be reasons to do a larger number of tests regarding a specific service over a given period of time, for instance if new services are launched.

Event data do not provide information of a time interval, but just a number of results, each for a corresponding point in time. To allow statements about time periods, it is either possible to use all original data points for defining and aggregating parameters for each time interval of interest, or as an alternative, relatively small time intervals have to be defined for which a first, "basic", aggregation step is carried out which then allows higher aggregations independent of the original data.

8.2.2 Key Performance Indicators / Parameters

A particular feature of a Key Performance Indicator (KPI or parameter) - in comparison to raw data - is given by the fact that KPIs are defined in an abstract manner, thereby allowing a common interpretation for parameters computed from different data sources and on various aggregation levels. Usually, there are two possible reasons for a parameter to be identified as a KPI, either:

- the KPI is a function aggregation of different parameters; or
- the KPI represents a very important quality measure related to the customer's perspective.

In the latter case, data aggregation is not necessarily implied.

Parameters are defined to serve specific purposes like answering questions about QoS or performance by utilizing raw data or basic aggregations of raw data. This might also include combinations of different data by mathematical operations like ratios. Unlike raw data, parameters are independent of the underlying software releases or system vendors. One could also define them as being independent of different underlying systems, if appropriate.

Parameters based on performance data are for instance the Cut-of-Call Ratio, which is based on two different NE counters, namely the number of unintentionally terminated calls divided by the number of successful call attempts times 100 %. Data from active probing systems allow the definition of parameters like the Recharging-Failure-Ratio, SMS-E2E-Failure-Ratios and so on.

For the definition and computation of parameters, rules for handling missing data are needed. Therefore methods for data substitution become a major point when talking about parameter computation and aggregation and will be covered in some detail after defining aggregation hierarchies of interest.

8.3 Aggregation hierarchies

Aggregation hierarchies are commonly divided into temporal and spatial aggregation hierarchies, where temporal in fact refers to different time intervals while the term spatial may also cover aggregations over items with similar properties with respect to an attribute of interest.

8.3.1 Temporal aggregation

Temporal aggregation levels that should be used for a given parameter will depend on the intended use of the parameter as well as on the raw data frequency. For most parameters, sensible aggregations levels will be some or all of the ones given in the following set:

- quarter-hour;
- hour;
- day;
- week;
- month;
- quarter year;
- calendar year; and
- business year.

In addition, incomplete temporal aggregation levels can be defined, for instance cumulative values for the current week based on the daily values that are available so far. This is of particular interest for parameters that are based on ratios or mean values because these may be interpreted directly. For the interpretation of incomplete parameters that are based on sums, the number of covered time units has to be taken into account.

8.3.2 Spatial aggregation

Spatial aggregation originally refers to aggregation levels from the smallest possible units like radio cells (or even sectors) in mobile communications up to the entire network. This can be done from a technical point of view for instance by taking the order "cell - BSC - MSC - (branch) - entire network", or from a regional point of view by "cell - state / region - entire network".

As mentioned before, the term "spatial" may also be used in the context of summarizing cells with similar properties regarding an attribute of interest, like all cells that cover motorways, or the position of a cell in terms of the surrounding area, whether it belongs to a large city, a small town or a rural area. In these cases, spatially incoherent units are aggregated.

8.4 Parameter estimation methods

The ideal situation of full information is rarely met in practice. Network elements or branches failing to deliver data in time are common reasons for missing data. Since in most situations, missing values as parameter values are unacceptable, even if parts of the raw data are missing, data estimation methods are needed. Depending on the situation, projection, substitution or combined estimation methods are suitable.

8.4.1 Projection method

The easiest method of data substitution is to project the available data to the entire time interval of interest. For example, if a fraction of 90 % of the expected data measuring the quality of a specific service within one hour is available and an hour-value is sought, these 90 % of the data are viewed as being representative for the hour of interest. If the aggregation contained of cumulating the entries, the value achieved by the available data has to be multiplied by a factor 100/90. If aggregated values are mean (or median) values, the mean (or median) of the available data is used as an aggregated value. If minimum or maximum values (or other quantiles besides the median) are sought as aggregated values, more sophisticated estimation methods should be applied like maximum likelihood methods.

Provided that a high percentage of data is available and there are no systematic reasons for missing data, the above procedure is sufficiently reasonable. However, an analysis of the times and circumstances of missing data might be of interest to identify the reasons, if missing values are becoming more frequent or appear suspect that there might be an underlying pattern.

If only a low percentage of data is available for a time period of interest, for instance less than 50 %, the above projection procedure is more than questionable. In particular if the parameter of interest is subject to dynamic changes over time, the results may be heavily biased. As an example consider usage data where a high percentage of missing data consists of records that should have been taken at night. If the usage is then estimated by projecting the available results, the usage is extremely overestimated. Therefore it seems sensible to define a "critical percentage" of data that need to be available for applying the method above. This percentage should depend on the specific service which is evaluated and on the needed precision of the aggregation results.

8.4.2 Substitution method

If the estimation of parameters can or should not be based on the available data, since a large number of data is missing or the data are not credible for some reason, substitution methods apply values from former time periods. This can either be done by using the last available value for the parameter of interest, which is only sensible for static parameters that are not subject to dynamic changes over time, or by using the last available value of a comparable time period with similar properties like the same day-type (weekday/Saturday/holiday or Sunday) or the same day of the week and the same time.

8.4.3 Application of estimation methods

Common problems that complicate the application of the methods suggested above are given by:

1) Unavailability of reference data: The number of missing data is needed for deciding which method should be used and for the application of the projection method.

2) Determination of values for substitution: Comparable time intervals have to be defined and substitution values may be stored in a data base, which needs to be updated and maintained. In addition, calendar information about holidays/working days, weekdays and so on is needed.

3) Parameters that are defined as ratios: Either numerator and denominator are estimated separately based on the available information for each part, or the ratio is estimated as a whole by using only data with information about both, numerator and denominator. In the situation of full information, there is no difference between both possibilities, in case some data are available for one part of the ratio and not for the other, both strategies yield different results.

Referring to data aggregation, the question arises, at which aggregation level an estimation should take place. Is it acceptable to use estimated values as a basis for higher aggregations? Data aggregation procedures combining both introduced methods are derived in the following, originating from a summary of desirable attributes of aggregation procedures.

8.4.4 Attributes of aggregation operators

Aggregation methods may be evaluated according to the following attributes:

1) The result should be meaningful, that is as near as possible to the true value of interest. In particular, NULL-values are not sensible as a result of a higher level aggregation. In addition, all information about missing values should be used to take non-available data into account, to avoid biased parameter values. (Moreover, the variance of parameter values caused by estimation methods should be as small as possible.)

2) Aggregation results should be reproducible and understandable. In particular, at higher aggregation levels no estimation procedure should be used so that results on a higher aggregation level are in accordance with values of the underlying level.

3) Aggregation results should not depend on the used aggregation paths, i.e. there should be no difference of results, if spatial or temporal aggregation steps are interchanged as well as direct aggregation and aggregation with intermediate steps should not lead to different results. Independence of paths refers to aggregation calculations.

4) Results should be consistent. On a given aggregation level, individual aggregation results should agree with total result, i.e. the sums for different branches should add up to the total sum for the company and so on. Consistency refers to aggregation results.

5) The applied calculation procedures should be rather easy. This also implies independence of past values like those from previous time periods.

6) Independence of network references like assignment of results to network elements.

In general, it is not possible to meet all requirements at the same time. Easy methods may lead to non-sensible results, while methods that contain estimation procedures often rely on values from previous time periods or network references and may be more sophisticated. In particular the requirements 1 and 5/6 are contradicting as estimation methods ignoring network references and past values will presumably often lead to worse results compared to methods that take into account all available information.

One idea to combine the above requirements to a certain extent is to define a smallest temporal and spatial aggregation level for which the data are completed by estimation procedures (for missing parameter data) or basic aggregations (for event data and parameter data with different lowest levels), like per hour per cell. This yields an equally spaced "complete" data pool and therefore simplifies all further aggregation steps and in particular ensures consistency of results and independence of aggregation paths.

One major disadvantage of this method is the fact that estimation procedures have to be applied on low aggregation levels which rely heavily on reference data and good substitution values or projection procedures. For parameters that are dynamic over time, time series methods as covered in clause 7 should be considered, which then implies more complicated calculation procedures for low aggregation levels and therefore might take some computation time.

8.5 Weighted aggregation

In many situations, in particular if the QoS perceived by the customer is of interest, simple aggregations of the available information or estimated values are not very meaningful. A better approach would be to take into account e.g. how many users are affected if a service fails. That leads to the idea of weighted aggregation, where for instance the usage can be applied for weighting, respectively. It should be noted, however, that weighted aggregation methods will in general lead to non-consistent results in the sense of property 4 from clause 8.4.4.

8.5.1 Perceived QoS

Depending on the point of view and the corresponding intention of a parameter of interest, it appears reasonable to only consider users view instead of the network view e.g. by taking the usage of a service into account. Depending on the applied aggregation procedure, this may have already been done implicitly. For instance, if the aggregated cut-of-call-ratio for a particular week is considered, different aggregation procedures imply different weightings (it is assumed that values are available hourly):

1) If the cut-of-call-ratio is stored for hourly intervals and the weekly aggregation is done by averaging all values for the week of interest, no weighting is carried out and each hour viewed as being equally important. This does not correspond with the users perspective. (Note that a geometric rather than an arithmetic mean should be applied for averaging ratios.)

2) If the numerator and denominator are stored for hourly intervals and the weekly aggregation is done by first summing all entries for the numerator and denominator separately and then dividing both numbers, an implicit weighting is carried out. Since high usage time intervals contribute a larger number of call attempts than low usage intervals, thereby the users perspective is expressed.

When applying the first method, one should consider using weighted means instead of simple means. Depending on the type of parameter, weighted arithmetic means are computed according to:

$$\bar{x}_w = \sum_{i=1}^{n} x_i w_i$$

where $\sum_{i=1}^{n} w_i = 1$, weighted geometric means are given by:

$$\tilde{x}_w = \left(\prod x_i w_i\right)^{1/n}$$

where $\prod w_i = 1$. (For unweighted arithmetic or geometric means, all weights are given by $w_i \equiv 1/n$ or $w_i \equiv 1$, respectively.)

Weights can either be based on true or average usage curves, but also on strategic reasons or any other procedure in accordance with the aim of the analysis. An average usage curve may for instance be achieved by averaging over the past 30 working days, the past 5 Saturdays and the past 5 Sundays or holidays or by applying some kind of time series modelling and forecasting methods. Weights based on usage curves are then computed as:

$$w_i = \frac{u_i}{\sum u_i} \text{ or } w_i = \frac{u_i}{\prod u_i}$$

respectively, where u_i is the true or estimated usage within time period i.

If the second method from above is applied, weighting is done implicitly with the actual usage curve. However, other problems arise in particular regarding missing data handling as mentioned in clause 8.4.3. For each time period, the percentage of missing data might be of interest for applying projection or substitution methods and to ensure that the cut-of-call-ratio does not exceed 1, e.g. the number of unintentionally terminated calls should not exceed the number of successful call attempts, one might only want to consider data pairs where both numbers are known. When using the first method, this could be avoided by estimating only on an hourly basis.

Remark: For ratios, higher level aggregations are commonly achieved by applying the second method because of the implicit weighting, which is more intuitive.

Data from active probing systems are generally not weighted implicitly, since probing frequencies are commonly non-representative for customer behaviour. In this context, the idea of weighting might even be of importance in more than one respect.

1) Since data from active probing systems are not equally spaced, a weighting of each trial result by the time between two trials in some way could be considered. This can either be realized by defining (rather small) time intervals for which all trials done within this interval are summarized without weighting or alternatively by computing using half of the time interval between the last trial and the current one and half of the time interval between the current trial and the next one as a basis for weighting. If such weighting is considered, an upper bound for the defined underlying intervals should be considered and strategies for the situation that the active probing system does not work or data are not delivered for a longer time period are to be thought of (estimation or NULL-values, depending on the situation and parameters of interest).

2) A second - and probably more important - way of weighting results from active probing system is the usage weighting for achieving the perceived QoS as explained before.

If both types of weighting are applied, combined weights are computed as $w_i = \frac{u(t_i)}{\sum u(t_i)}$, where $u(t_i)$ is the usage within time period t_i assigned to probing trial i according to the distribution of trials over time (1st weighting), either for a basic aggregation level for further aggregation or for the desired aggregation level directly.

8.5.2 Weighted quantiles

For duration values as results from active probing trials, quantiles represent meaningful aggregation results on higher aggregation levels. From the above weighting discussion the necessity of determining weighted quantiles arises. Due to the calculation of quantiles based on ordered data values, a weighting similar to those for mean values is not applicable. Instead, a **replication** algorithm could be used for computing weighted quantiles. This algorithm simply repeats each value according to an assigned weight and calculates the desired quantile of the resulting new data set. (If weights are irrational, sensible rounding is needed.)

EXAMPLE: The original (ordered) data set of 10 MMS-E2E delivery times is given by 51, 55, 60, 61, 65, 70, 71, 72, 72, 80 seconds. These measurements have been taken at different daytimes and therefore get a weight of 1, if taken at night, 2, if taken in the morning or late in the evening and 4, if taken between noon and 8 p.m. for instance. According weights are therefore given by 1, 4, 2, 2, 1, 4, 4, 2, 1, 4 resulting in a data set with 25 data: 51, 55, 55, 55, 55, 60, 60, 61, 61, 65, 70, 70, 70, 70, 71, 71, 71, 71, 72, 72, 72, 80, 80, 80, 80. Quantiles from the original and the replicated data set will in general lead to slightly different results.

If a weighting according to some kind of usage curve is aimed, this curve can be used as a replication function or replication curve and represent the basis for defining the needed weights. To simplify the computation, weights may be defined by identifying the minimum of the replication function $r_{\min}$ and to define the weights according to:

$$w_i = round\left(r_i \big/ r_{\min}\right).$$

If a uniform concept for weighting of any kind of parameter is sought, the approach based on replication functions might also be used for means or non-accessibilities or other parameters of interest. Differences for instance between conventionally weighted means and means weighted by replication curves are only due to the applied rounding step for the latter approach.

8.6 Additional data aggregation operators

In the following, some additional data aggregation operators are covered, adding to those mentioned in clause 8.1, that are in some sense individual regarding their attributes and/or application.

8.6.1 MAWD and BH

In particular for network maintenance an aggregation operator of interest is the "Monthly Average Working Day" (MAWD). This operator can be viewed as being an aggregation result as well as a possible weighting function for other aggregation operators.

The monthly average working day of a given data set is computed by filtering all data from working days within a given month first and then computing the mean value for each hour of the day over all data from corresponding hours. The result is therefore given by a vector of 24 entries, each corresponding to one hour of the day (0 h - 1 h, ..., 23 h - 24 h).

Based on the MAWD, the "Busy Hour" (BH) is defined as the hour in which the MAWD-vector takes its maximum value. In mathematical notation, this is *argmax(m)*, where $m = (m_1, ..., m_{24})^T$ is the vector resulting from applying the MAWD operator.

8.6.2 AVGn

The class of parameters AVGn is applied for similar reasons as the BH-operator mentioned in clause 8.6.1. Both try to identify peaks of high usage or traffic, where the BH-operator considers hours where the highest usage is observed on average, while the AVGn-operators are interested in the maximum usage or traffic for a given calendar week. The mean of the *n* largest values realized on *n* different days (*n* between 1 and 5 or 7, depending on intended use) is defined as AVGn.

9 Assessment of performance indices

9.1 Estimation of performance parameters based on active service probing systems

End-to-end service probing systems yield valuable information about services and systems that may not be provided by the network elements alone. Active probing systems are used to view the customer perspective of the Quality of Service, i.e. the perceived QoS. Typical parameters that may be computed based on active probing systems are failure ratios, accessibilities and end-to-end-delivery times for a number of different services.

One characteristic of active probing systems is that the tests are often done more or less equally distributed over time for utilizing the equipment as exhaustingly as possible. In this respect they fail to reflect the customers perspective, since derogation during the day will be a lot more severe than after midnight due to lower volume of traffic for almost all services at night.

From a statistical point of view, end-to-end active probing systems try to estimate the real behaviour of a service by taking a defined number of samples. Therefore, the results of measurement campaigns have to be interpreted as the current view on a certain service and need not necessarily represent customer experience. Depending on the number of considered samples, the connection between observed measurement results and unknown real behaviour may vary.

9.2 Monitoring concepts

To ensure that network problems are noticed and remedied as quickly as possible, monitoring concepts based on active probing results are important tools for an efficient alarming system. Such monitoring tools may be based on control charts or other alarming rules.

9.2.1 Control charts

Control charts are based on the assumption that if the service under study works properly, data gained from active probing systems follow a stable distribution with given parameters. From former observations, the underlying distribution may be identified and parameters have to be estimated. Control charts are now set up based on statistical considerations such that in case of network problems, i.e. the process is "out of control", an alarm is created. On the other hand, false alarms have to be avoided, that is as long as the process is "in control", no alarming should occur.

Control charts generally visualize the development of a quality characteristic of interest over time similar to a line diagram as shown in figure 9.1. Further, a target line and control and or warning limits are added to the plot. The target line represents the line around which data values are expected. Warning and control limits may be used to define identify realizations indicating that the process it "out of control". Different types of control charts were invented for different types of data.

9.2.1.1 Shewhart control charts

If data are normal or mean values are considered (central limit theorem, compare section), Shewhart charts for normal data may be applied. In this case, the current short-term data is compared against an underlying data model which represents the long-term behaviour. According to this model it is possible to define the usual or "normal" situation. This is required to pay attention to unusual situations. Shewhart control charts are widely used in different sections of the industry.

9.2.1.2 CUSUM and EWMA charts

Two other approaches can be used to introduce some kind of weighting into control charts. The CUSUM approach uses sums data up over time and therefore indicates the behaviour over a greater period of time. A slightly different approach is represented by "exponentially weighted moving average" charts where older values gain less influence than newer data does.

9.2.2 Other alarming rules

Furthermore, the deviation between the long-term data model and the short-term monitoring data should lead to consecutive actions if a critical state is reached. This relation is defined as "alarming rules". One example for alarming rules is the set of the Western Electric Rules.

9.3 Methods for evaluation of objectives

Commonly objectives are formulated in terms of target values for important parameters. Then the evaluation of objectives could mean to assess to which extend these aims have been achieved within a given time-period (i.e. month or business year). If there is only one important parameter, this is a rather easy task. However, if a number of pre-defined parameters are to be combined in an overall measure and in addition different groups (i.e. branches or departments) are to be compared regarding their performance, the main issue for evaluation will be to define a common measurement scale for all parameters. This allows the combination to an overall evaluation index of some kind and thereby a comparison of groups is facilitated.

In the following, two methods are described that allow the evaluation of objectives, namely the desirability approach and the loss function approach. Both approaches rely on definitions of target values and specification limits for the parameters. In this context, parameter values are denoted by $y_i, i = 1,...,P$ and target values are denoted by $\tau_i, i = 1,...,P$. Specification limits are given as upper and / or lower specification limits USL_i, LSL_i for each parameter under consideration $i = 1,...,P$. (It might also be sensible to consider lower and upper target values, if the target is given as an interval instead of a numerical value.)

9.3.1 Desirability functions

Desirability functions use a transformations of the values y_i to the interval [0,1] based on system requirements by defining a transformation function based on target values and specification limits. Desirability functions are piecewise defined continuous linear functions where desirability values of 0 are assigned to parameter values y_i outside the specification limits, realizations on target get desirability values of 1 and outcomes between target and specification limits are assigned by a linear connection or a power transformation thereof.

The principle of desirabilities is best explained by providing example desirability functions as summarized in figure 9.1.

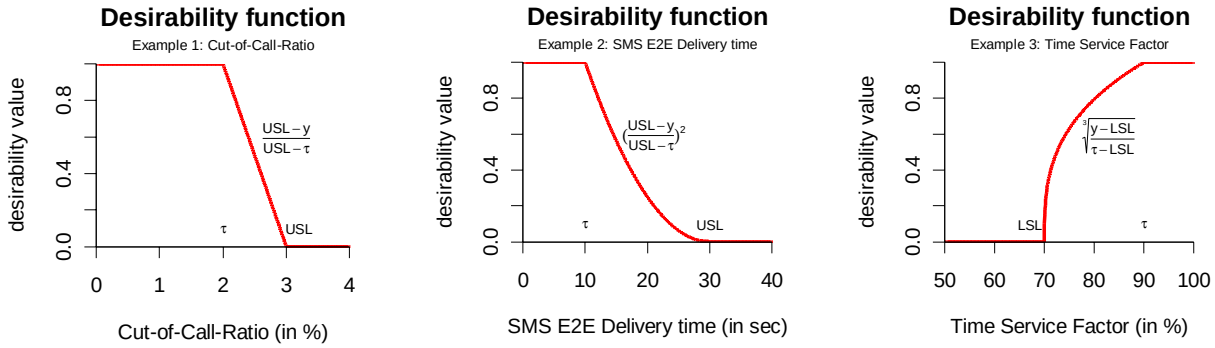


Figure 9.1: Examples of different desirability functions

9.3.2 Loss functions

Loss functions in contrary evaluate a realized parameter value in terms of costs arising due to deviation from target or realization outside the specification limits. Therefore values within the interval [0,∞) will be achieved. The main issue for the specification of loss functions is the assignment of arising costs. The loss of earnings if services are not fully usable may be stated rather easily, but quantifying the image loss and corresponding costs might be a much more difficult task.

For each parameter of interest, the arising loss for a value y_i is given by $L(y_i) = c(y_i - \tau_i)^2$ or alternatively $L(y_i) = c \min\left((y_i - USL_i)^2, (y_i - LSL_i)^2\right)$, where $c \in IR$ quantifies the arising cost.

Mainly, normal distributed values are in the focus if loss functions are discussed. Generally speaking, the area which is covered by the lower and upper tails of the normal (or Gaussian) distribution is in the main interest. These branches violate the guaranteed specification levels Upper Specification Level (USL) for the upper and Lower Specification Level (LSL) for the lower tail. All values in these areas represent defects referring to the observed process. The underlying theory specifies rules how to set the limits and how to proceed with asymmetric cases. One useful hint for further research in this area is the "six sigma approach" which is wide-spread in the industry.

Annex A (informative): Examples of statistical calculations

In the following, some example computations are given for different topics. All computations are done step by step as well as by applying statistical software tools. The statistical software mainly applied here is the open source language and environment R, based on the S language which is widely spread in the statistics area. For further information and download, the reader is referred to:

- http://www.r-project.org

R is a piece of freely distributed software, its installation is straightforward and commonly done within five minutes. For further applications and more sophisticated statistical methods, a number of libraries is available from the website. For creating graphics and first steps in programming see also [VEN] (see bibliography). For reliable results, the use of R is highly recommended.

As Excel is a standard software used for calculations, also some commands are given for Excel-users. Nevertheless, it must be said that Excel does not have a uniform definition for the computation of different expressions or operators, e.g. quantiles. Most of the mathematical functions are defined only with regard to specific desired tests. The user should therefore be warned to use any of Excels mathematical procedures without a deeper understanding of the functionality differences between these procedures.

A.1 Confidence intervals for binomial distribution

This example tries to clarify the usage of the Pearson-Clopper formula which is related to the binomial distribution and may be used for measurements with a small amount of measurement data.

EXAMPLE: During a one hour manual test of service X the service access attempts lead to the following results ("0" represents an unsuccessful attempt, "1" a successful attempt).

No. 1-10	1	0	1	0	1	1	1	1	0	0
No. 11-20	0	1	1	1	1	0	1	1	0	1
No. 21-30	1	1	0	1	1	1	1	1	1	1
No. 31-40	0	1	0	0	1	1	1	1	1	1

Within the $n = 40$ attempts $m = 29$ have been successful. The point estimation leads to $p = \frac{m}{n} = \frac{29}{40} = 0{,}725$.

A.1.1 Step by step computation

Since the results show a binary outcome, the binomial distribution can be applied in any case. At first, the allowance of the easier to handle Normal distribution has to be checked via the following expression:

$$n \cdot p \cdot (1 - p) = 7{,}975 < 9$$

Therefore, the Normal distribution should not be used for this measurement. Furthermore, the following computations are directly based on the binomial distribution.

If the required confidence level is defined as $1 - \alpha = 0{,}95$, the resulting α value is $\alpha = 0{,}05$. According to this, the Pearson-Clopper formulas now read:

$$p_1 = \frac{m \cdot F_{2m,2(n-m+1);\frac{\alpha}{2}}}{n - m + 1 + m \cdot F_{2m,2(n-m+1);\frac{\alpha}{2}}} = \frac{29 \cdot F_{58,24;0{,}025}}{12 + 29 \cdot F_{58,24;0{,}025}}$$

$$p_2 = \frac{(m+1)\cdot F_{2(m+1),2(n-m);1-\frac{\alpha}{2}}}{n-m+(m+1)\cdot F_{2(m+1),2(n-m);1-\frac{\alpha}{2}}} = \frac{30\cdot F_{60,22;0,975}}{11+30\cdot F_{60,22;0,975}}$$

Eventually, four steps have to be executed to get the relevant confidence interval:

1) Lookup if the needed quantile values of the F distribution are tabulated.

2) If the quantiles are not tabulated, try the relation $F_{n_1,n_2;1-\gamma} = \frac{1}{F_{n_2,n_1;\gamma}}$ to get the required information.

3) If both approaches do not succeed, try the approximation $F_{n_1,n_2;\gamma} \cong \exp(u\cdot a - b)$ for γ in the range $0{,}5 < \gamma < 1$.

4) Determine the confidence interval by using the quantile values.

Now, the quantiles $F_{58,24;0,025}$ and $F_{60,22;0,975}$ have to be retrieved before the Pearson-Clopper values are computable.

1) Looking up some tabulated quantile values may lead to the following results:

$$F_{60,22;0,975} = 2{,}145$$

If the quantile $F_{58,24;0,025}$ can not be found, the following steps may be appropriate.

2) If $F_{58,24;0,025}$ is missing in the tables, perhaps the quantile

$$F_{24,58;0,975} = \frac{1}{F_{58,24;0.025}}$$

is available. If this is also not the case, a first sight approximation is given by a neighbouring quantile value:

$$F_{24,60;0,975} = \frac{1}{F_{60,24;0,025}} = 1{,}882$$

$$\Rightarrow F_{60,24;0,025} = \frac{1}{F_{24,60;0,975}} = 0{,}5313$$

3) Since the quantile variable $\gamma = 0{,}975$ lies in the range $0{,}5 < \gamma < 1$, the approximation

$$F_{n_1,n_2;\gamma} \cong \exp(u\cdot a - b)$$

can be applied. Therefore, the following computational steps have to be executed to determine $F_{24,58;0.975}$ in a more precise way:

- At first, the parameter d is done:

$$d = \frac{1}{n_1 - 1} + \frac{1}{n_2 - 1} = \frac{1}{24-1} + \frac{1}{58-1} \approx 0{,}06102$$

- Before computing $c = \frac{(u_\gamma)^2 - 3}{6}$, the 0,975-quantile of the standard normal distribution $N(0,1)$ has to be retrieved from a table:

$$\gamma = 0{,}975 \Rightarrow \quad u_{0{,}975} = 1{,}96$$

So c reads

$$c = \frac{(1{,}96)^2 - 3}{6} = 0{,}14026$$

- As a result, b is given by:

$$b = 2 \cdot \left(\frac{1}{n_1 - 1} - \frac{1}{n_2 - 1} \right) \cdot \left(c + \frac{5}{6} - \frac{d}{3} \right) = 2 \cdot \left(\frac{1}{24 - 1} - \frac{1}{58 - 1} \right) \cdot \left(0{,}14026 + \frac{5}{6} - \frac{0{,}06102}{3} \right) = 0{,}04944$$

- With these results, a leads to:

$$a = \sqrt{2d + cd^2} = \sqrt{2 \cdot 0{,}06102 + 0{,}14026 \cdot 0.06102^2} = 0{,}35$$

- Finally, the approximation for the quantile value reads:

$$F_{58,24;0,975} \cong \exp(1.96 \cdot 0.35 - 0.04944) = 1.8899$$

The originally searched quantile value $F_{58,24;0,025} = \frac{1}{F_{24,58;0.975}}$ results then in:

$$F_{58,24;0,025} = \frac{1}{1{,}8899} = 0{,}5291$$

4) After the quantiles of the F distribution are known, in the last step the Pearson-Clopper values itself can be determined:

$$p_1 = \frac{m \cdot F_{2m,2(n-m+1);\frac{\alpha}{2}}}{n - m + 1 + m \cdot F_{2m,2(n-m+1);\frac{\alpha}{2}}} = \frac{29 \cdot F_{58,24;0,025}}{12 + 29 \cdot F_{58,24;0,025}} = \frac{29 \cdot 0{,}5291}{12 + 29 \cdot 0{,}5291} = 0{,}5611$$

$$p_2 = \frac{(m+1) \cdot F_{2(m+1),2(n-m);1-\frac{\alpha}{2}}}{n - m + (m+1) \cdot F_{2(m+1),2(n-m);1-\frac{\alpha}{2}}} = \frac{30 \cdot F_{60,22;0,975}}{11 + 30 \cdot F_{60,22;0,975}} = \frac{30 \cdot 2{,}145}{11 + 30 \cdot 2{,}145} = 0{,}854$$

With these values, the confidence interval for the given measurement can be described as:

$$[p_1; p_2] = [0{,}5611; 0{,}854]$$

A.1.2 Computation using statistical software

The different calculations can be executed by R. To enable a user-oriented simplicity, the according expressions are given in the next clauses.

A.1.2.1 Computation in R

Required quantiles of the F-distribution may also be obtained in R. Here no approximation is carried out. Commands (marked by " >") and results (marked by "[1..]") are given by:

```
> qf(0.025, 24, 58)
[1] 0.4797205

> qf(0.975, 60, 22)
[1] 2.144594
```

Alternatively, a function can be defined for computing the Pearson-Clopper confidence interval directly. This function takes the following input variables:

- n: Number of trials (total);
- m: Number of (un-)successful interesting trials, either successful ones or non-successful ones;
- alpha: desired confidence level (default: 5 % 1- alpha = 95 %, means alpha = 5 % = 0,05).

The output consists of:

- estimator: The estimated value of the according rate parameter;
- confidence interval for the estimator (lower and upper bound).

The function is given by:

```
pearson.clopper <- function(n, m, alpha = 0.05) {
    # computation of F-quantiles
    f1 <- qf(alpha/2,2*m,2*(n-m+1))
    f2 <- qf(1-alpha/2,2*(m+1),2*(n-m))
    # computation of confidence limits
    p1 <- m * f1/(n-m+1+m*f1)
    p2 <- (m+1)*f2/(n-m+(m+1)*f2)
    out <- list(estimator = m/n, confidence.interval = c(p1, p2))
    return(out)
}
```

The function is applied by calling it with the required arguments. The result for the above example is given by:

```
> pearson.clopper(40, 29)
$estimator
[1] 0.725

$confidence.interval
[1] 0.5611171 0.8539910
```

A.1.2.2 Computation in Excel

In Excel, quantiles of the F-distribution are derived by applying the functions:

FINV(p-value;df1;df2)

Related to the use of Excel, it is very important to have a very clear understanding what is done by a certain expression. For example, the calculation of FINV depends on the parameter p-value which is NOT the same as the parameter "alpha" in the R section!

A.2 Transition from binomial to normal distribution

To use of the transition from a binomial distribution to a normal one, the condition:

$$n \cdot p \cdot q = n \cdot p \cdot (1 - p) \geq 9$$

has to be fulfilled.

EXAMPLE 1: If $n = 30$ samples are gathered which lead to an estimated rate of $p = 0{,}8$, the condition reads:

$$n \cdot p \cdot q = n \cdot p \cdot (1 - p) = 30 \cdot 0{,}8 \cdot 0{,}2 = 4{,}8 < 9$$

This means, the approximation is not allowed and confidence intervals have to be calculated with the Pearson-Clopper formula.

EXAMPLE 2: Now, the same rate p = 0,8 is estimated on a basis of n = 300 samples. The according relation reads

$$n \cdot p \cdot q = n \cdot p \cdot (1 - p) = 300 \cdot 0{,}8 \cdot 0{,}2 = 48 > 9$$

In this case, the approximation of the binomial distribution by a normal distribution is allowed. The confidence intervals can be calculated with the according expressions of the normal distribution.

A.3 Definitions of EG 201 769

The following clause presents another definition of confidence intervals related to the normal distribution. It can be found in EG 201 769-1 [1]:

- Relationship between the accuracy of estimator of the unsuccessful call ratio and the number of calls to be observed.

If k unsuccessful calls are observed out of N call attempts, then the true value of the unsuccessful call ratio lies between $\frac{k}{N} - \Delta$ and $\frac{k}{N} + \Delta$ with a confidence level $1 - \alpha$, Δ being approximated (for large value of N) by:

$$\Delta \approx \sigma(\alpha) \times \sqrt{\frac{p \times (1 - p)}{N}}$$

where p is the expected unsuccessful call ratio and $\sigma(\alpha)$ is the $(1 - \frac{\alpha}{2}) \times 100$ percentile of the normal distribution with mean 0 and standard deviation 1 (N(0,1)). I.e. the number of call attempts to be observed should be:

$$N = \frac{\sigma(\alpha)^2 \times p(1 - p)}{\Delta^2}$$

- If the confidence level is $1 - \alpha = 0{,}95$ then $\sigma(\alpha) = 1{,}96 \approx 2$.
- If the required accuracy for $p \leq 0{,}01$ is $\Delta = 0{,}001$, then the number of call attempts to be observed should be $N = 4 \times 10^6 \times p(1\text{-}p)$ for a confidence level of 95 %.
- If the required accuracy for p>0,01 is $\frac{\Delta}{p} = 0{,}1$, then the number of call attempts to be observed should be $N = 400 \times ((1 - p)/p)$ for a confidence level of 95 %.
- For example, if the expected unsuccessful call ratio is 1 %, the number of call attempts to be observed should be

  $$N = 4 \times 10^6 \times 0{,}01(1 - 0{,}01) = 39\,600$$

  for an accuracy of Δ = 0,001 with a confidence level of 95 %.
- If the unsuccessful call ratio is expected to be 3 %, then the number of call attempts should be:

  $$N = 400 \times ((1 - 0{,}03)/0{,}03)\ 13\,000$$

  for a relative accuracy of $\frac{\Delta}{p} = 0{,}1$ and with a confidence level of 95 %

A.4 Calculation of confidence intervals

This clause gives more information about the calculation of confidence intervals. Due to the possibility that also small numbers may occur for example if service probing is done manually, the calculation of confidence intervals is based on the relations given by the Pearson-Clopper expressions.

The structure of this clause is as follows:

- Starting with some visualized use cases of the Pearson-Clopper formulas, an impression of the relationship between the estimated rate value and the according confidence interval is given.

A.4.1 Estimated rate 5 %

The confidence interval gets smaller with an increasing number of available samples. The less data is available, the higher the uncertainty is. Another effect which is covered by the Pearson-Clopper approach is the asymmetric behaviour of the upper and lower limits of the confidence interval. Additionally, this asymmetricy depends on the estimated rate values (see following graphs).

Some further remarks might be helpful:

- The confidence interval can be calculated for rather small sample sizes.
- An overestimation like it would have appeared by applying the normal (Gaussian) approximation is not recognizable.
- If a rate value is equal to 0 %, this is also the value of the lower limit of the confidence interval. The calculation of quantiles of the F-distribution is not valid in this case.
- If a rate value is equal to 100 %, this is also the value of the upper limit of the confidence interval. The calculation of quantiles of the F-distribution is not valid in this case.

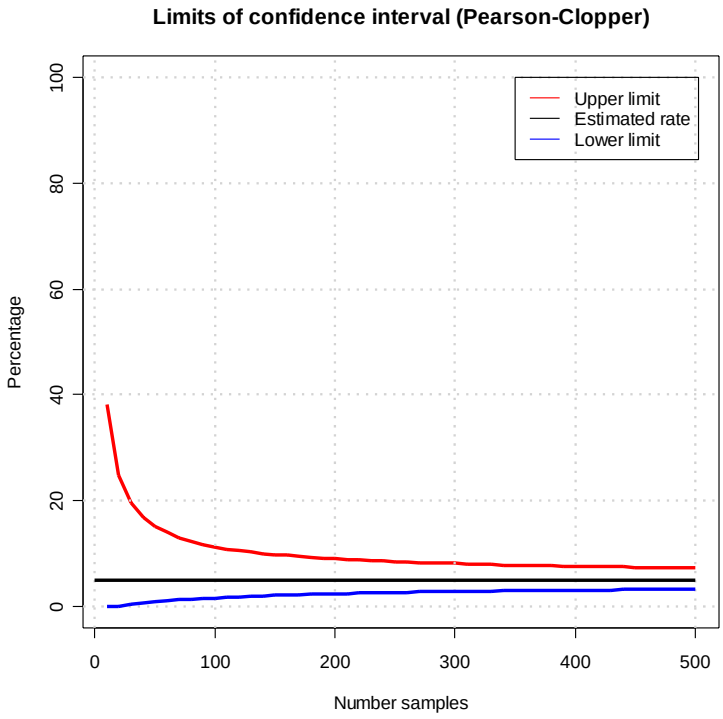


Figure A.4.1: Confidence interval for estimated rate of 5 %

The depicted limit curves can be found in the columns of the following tables (estimated rate is constant, number of measurements varies).

A.4.2 Estimated rate 50 %

In figure A.4.2 the confidence interval for an estimated rate of 50 % is depicted. In this case the confidence interval owns a symmetric behaviour.

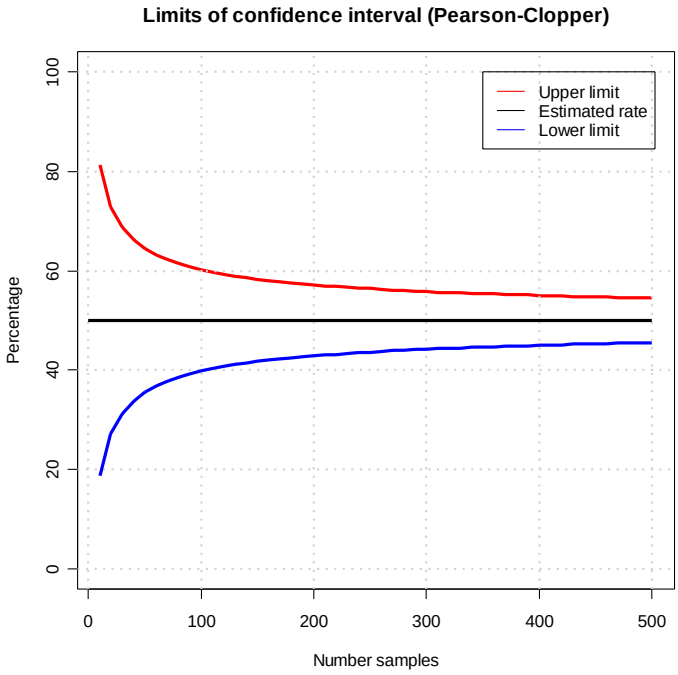


Figure A.4.2: Confidence interval for estimated rate of 50 %

A.4.3 Estimated rate 95 %

Figure A.4.3 describes the situation according to a 95 % rate. The situation is comparable with the graph of the 5 % rate.

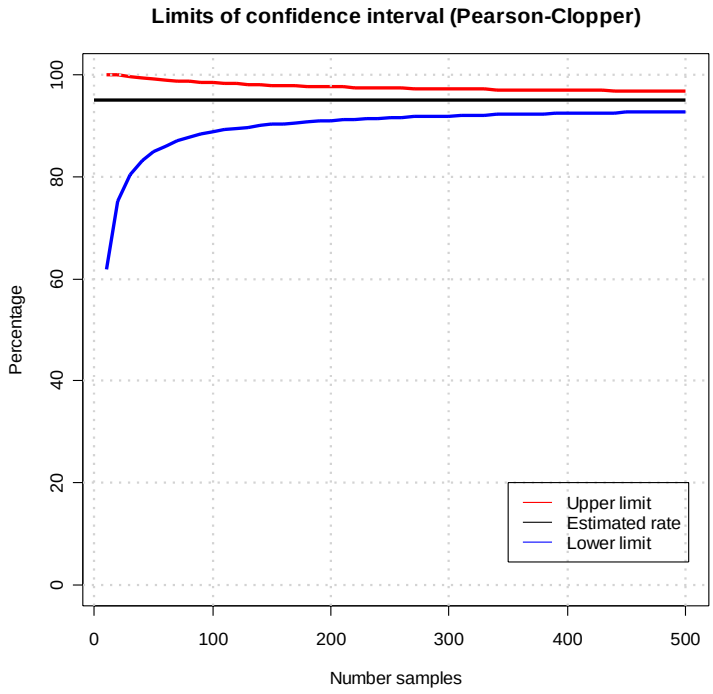


Figure A.4.3: Confidence interval for estimated rate of 95 %

A.4.4 Lower limit of confidence intervals according to Pearson-Clopper formula

Table A.4.1 contains values which specify the lower limit of the confidence interval. The lower limit depends on the number of samples and the according rate value. In figures A.4.1 to A.4.3 this information can be found at the blue lines.

Table A.4.1

	Rate																				
NrMeas	1 %	5 %	10 %	15 %	20 %	25 %	30 %	35 %	40 %	45 %	50 %	55 %	60 %	65 %	70 %	75 %	80 %	85 %	90 %	95 %	100 %
100	0,03 %	1,64 %	4,90 %	8,65 %	12,67 %	16,88 %	21,24 %	25,73 %	30,33 %	35,00 %	39,80 %	44,70 %	49,72 %	54,82 %	60,02 %	65,29 %	70,74 %	76,37 %	82,24 %	88,48 %	95,15 %
200	0,12 %	2,42 %	6,22 %	10,35 %	14,69 %	19,16 %	23,74 %	28,41 %	33,15 %	37,96 %	42,86 %	47,82 %	52,85 %	57,95 %	63,14 %	68,38 %	73,75 %	79,25 %	84,93 %	90,91 %	97,53 %
300	0,21 %	2,83 %	6,85 %	11,16 %	15,62 %	20,20 %	24,87 %	29,61 %	34,41 %	39,28 %	44,19 %	49,17 %	54,21 %	59,31 %	64,46 %	69,69 %	75,00 %	80,43 %	86,01 %	91,84 %	98,34 %
400	0,27 %	3,08 %	7,24 %	11,65 %	16,19 %	20,83 %	25,55 %	30,33 %	35,16 %	40,05 %	44,99 %	49,98 %	55,01 %	60,10 %	65,25 %	70,45 %	75,73 %	81,11 %	86,61 %	92,35 %	98,75 %
500	0,33 %	3,26 %	7,51 %	11,98 %	16,58 %	21,26 %	26,01 %	30,82 %	35,67 %	40,58 %	45,53 %	50,52 %	55,56 %	60,64 %	65,77 %	70,96 %	76,21 %	81,55 %	87,02 %	92,69 %	99,00 %
600	0,37 %	3,40 %	7,72 %	12,24 %	16,87 %	21,58 %	26,36 %	31,18 %	36,05 %	40,97 %	45,92 %	50,92 %	55,96 %	61,03 %	66,15 %	71,33 %	76,57 %	81,88 %	87,31 %	92,92 %	99,17 %
700	0,40 %	3,51 %	7,88 %	12,44 %	17,10 %	21,83 %	26,62 %	31,46 %	36,35 %	41,27 %	46,23 %	51,23 %	56,26 %	61,34 %	66,45 %	71,62 %	76,84 %	82,13 %	87,53 %	93,10 %	99,29 %
800	0,43 %	3,60 %	8,01 %	12,60 %	17,28 %	22,03 %	26,84 %	31,69 %	36,58 %	41,51 %	46,48 %	51,48 %	56,51 %	61,58 %	66,69 %	71,85 %	77,05 %	82,33 %	87,70 %	93,24 %	99,37 %
900	0,46 %	3,67 %	8,12 %	12,73 %	17,43 %	22,20 %	27,02 %	31,88 %	36,78 %	41,71 %	46,68 %	51,68 %	56,72 %	61,78 %	66,89 %	72,03 %	77,23 %	82,49 %	87,85 %	93,36 %	99,44 %
1000	0,48 %	3,73 %	8,21 %	12,84 %	17,56 %	22,34 %	27,17 %	32,04 %	36,95 %	41,88 %	46,85 %	51,85 %	56,89 %	61,95 %	67,05 %	72,19 %	77,38 %	82,63 %	87,97 %	93,45 %	99,50 %
1100	0,50 %	3,79 %	8,29 %	12,94 %	17,67 %	22,47 %	27,30 %	32,18 %	37,09 %	42,03 %	47,00 %	52,00 %	57,04 %	62,10 %	67,19 %	72,33 %	77,51 %	82,75 %	88,07 %	93,54 %	99,54 %
1200	0,52 %	3,84 %	8,36 %	13,03 %	17,77 %	22,57 %	27,42 %	32,30 %	37,21 %	42,16 %	47,13 %	52,13 %	57,16 %	62,23 %	67,32 %	72,45 %	77,62 %	82,85 %	88,16 %	93,61 %	99,58 %
1300	0,53 %	3,88 %	8,42 %	13,10 %	17,86 %	22,67 %	27,52 %	32,40 %	37,32 %	42,27 %	47,25 %	52,25 %	57,28 %	62,34 %	67,43 %	72,55 %	77,72 %	82,94 %	88,24 %	93,67 %	99,61 %
1400	0,55 %	3,92 %	8,48 %	13,17 %	17,93 %	22,75 %	27,61 %	32,50 %	37,42 %	42,37 %	47,35 %	52,35 %	57,38 %	62,44 %	67,52 %	72,64 %	77,81 %	83,02 %	88,31 %	93,72 %	99,64 %
1500	0,56 %	3,95 %	8,53 %	13,23 %	18,00 %	22,83 %	27,69 %	32,58 %	37,51 %	42,46 %	47,44 %	52,44 %	57,47 %	62,53 %	67,61 %	72,73 %	77,88 %	83,09 %	88,37 %	93,77 %	99,67 %
1600	0,57 %	3,98 %	8,57 %	13,28 %	18,07 %	22,89 %	27,76 %	32,66 %	37,59 %	42,54 %	47,52 %	52,52 %	57,55 %	62,61 %	67,69 %	72,80 %	77,95 %	83,15 %	88,42 %	93,81 %	99,69 %
1700	0,58 %	4,01 %	8,61 %	13,33 %	18,12 %	22,96 %	27,83 %	32,73 %	37,66 %	42,62 %	47,60 %	52,60 %	57,63 %	62,68 %	67,76 %	72,87 %	78,02 %	83,21 %	88,47 %	93,85 %	99,71 %
1800	0,59 %	4,04 %	8,65 %	13,38 %	18,17 %	23,01 %	27,89 %	32,79 %	37,73 %	42,68 %	47,66 %	52,67 %	57,69 %	62,75 %	67,82 %	72,93 %	78,08 %	83,26 %	88,52 %	93,89 %	99,72 %
1900	0,60 %	4,06 %	8,69 %	13,42 %	18,22 %	23,07 %	27,95 %	32,85 %	37,79 %	42,75 %	47,73 %	52,73 %	57,76 %	62,81 %	67,88 %	72,99 %	78,13 %	83,31 %	88,56 %	93,92 %	99,74 %
2000	0,61 %	4,09 %	8,72 %	13,46 %	18,27 %	23,12 %	28,00 %	32,91 %	37,84 %	42,80 %	47,78 %	52,79 %	57,81 %	62,86 %	67,94 %	73,04 %	78,18 %	83,36 %	88,60 %	93,95 %	99,75 %
2100	0,62 %	4,11 %	8,75 %	13,50 %	18,31 %	23,16 %	28,05 %	32,96 %	37,90 %	42,86 %	47,84 %	52,84 %	57,87 %	62,92 %	67,99 %	73,09 %	78,22 %	83,40 %	88,64 %	93,98 %	99,76 %
2200	0,63 %	4,13 %	8,78 %	13,53 %	18,35 %	23,20 %	28,09 %	33,01 %	37,94 %	42,91 %	47,89 %	52,89 %	57,92 %	62,97 %	68,04 %	73,13 %	78,27 %	83,44 %	88,67 %	94,00 %	99,77 %
2300	0,63 %	4,15 %	8,80 %	13,56 %	18,38 %	23,24 %	28,13 %	33,05 %	37,99 %	42,95 %	47,94 %	52,94 %	57,96 %	63,01 %	68,08 %	73,18 %	78,31 %	83,47 %	88,70 %	94,03 %	99,78 %
2400	0,64 %	4,16 %	8,83 %	13,59 %	18,42 %	23,28 %	28,17 %	33,09 %	38,03 %	43,00 %	47,98 %	52,98 %	58,01 %	63,05 %	68,12 %	73,22 %	78,34 %	83,51 %	88,73 %	94,05 %	99,79 %
2500	0,65 %	4,18 %	8,85 %	13,62 %	18,45 %	23,31 %	28,21 %	33,13 %	38,07 %	43,04 %	48,02 %	53,02 %	58,05 %	63,09 %	68,16 %	73,25 %	78,38 %	83,54 %	88,76 %	94,07 %	99,80 %
2600	0,65 %	4,19 %	8,87 %	13,65 %	18,48 %	23,35 %	28,24 %	33,16 %	38,11 %	43,07 %	48,06 %	53,06 %	58,09 %	63,13 %	68,20 %	73,29 %	78,41 %	83,57 %	88,78 %	94,09 %	99,81 %
2700	0,66 %	4,21 %	8,89 %	13,67 %	18,51 %	23,38 %	28,28 %	33,20 %	38,15 %	43,11 %	48,10 %	53,10 %	58,12 %	63,17 %	68,23 %	73,32 %	78,44 %	83,60 %	88,81 %	94,11 %	99,81 %
2800	0,67 %	4,22 %	8,91 %	13,70 %	18,53 %	23,41 %	28,31 %	33,23 %	38,18 %	43,15 %	48,13 %	53,13 %	58,16 %	63,20 %	68,26 %	73,35 %	78,47 %	83,62 %	88,83 %	94,13 %	99,82 %
2900	0,67 %	4,24 %	8,93 %	13,72 %	18,56 %	23,43 %	28,34 %	33,26 %	38,21 %	43,18 %	48,16 %	53,17 %	58,19 %	63,23 %	68,29 %	73,38 %	78,50 %	83,65 %	88,85 %	94,14 %	99,83 %
3000	0,68 %	4,25 %	8,95 %	13,74 %	18,58 %	23,46 %	28,36 %	33,29 %	38,24 %	43,21 %	48,19 %	53,20 %	58,22 %	63,26 %	68,32 %	73,41 %	78,52 %	83,67 %	88,87 %	94,16 %	99,83 %
3100	0,68 %	4,26 %	8,97 %	13,76 %	18,60 %	23,48 %	28,39 %	33,32 %	38,27 %	43,24 %	48,22 %	53,23 %	58,25 %	63,29 %	68,35 %	73,44 %	78,55 %	83,69 %	88,89 %	94,17 %	99,84 %
3200	0,68 %	4,27 %	8,98 %	13,78 %	18,63 %	23,51 %	28,42 %	33,35 %	38,30 %	43,27 %	48,25 %	53,26 %	58,28 %	63,32 %	68,38 %	73,46 %	78,57 %	83,71 %	88,91 %	94,19 %	99,84 %
3300	0,69 %	4,28 %	9,00 %	13,80 %	18,65 %	23,53 %	28,44 %	33,37 %	38,32 %	43,29 %	48,28 %	53,28 %	58,31 %	63,34 %	68,40 %	73,49 %	78,59 %	83,74 %	88,92 %	94,20 %	99,85 %
3400	0,69 %	4,29 %	9,01 %	13,82 %	18,67 %	23,55 %	28,46 %	33,40 %	38,35 %	43,32 %	48,31 %	53,31 %	58,33 %	63,37 %	68,43 %	73,51 %	78,61 %	83,75 %	88,94 %	94,21 %	99,85 %
3500	0,70 %	4,30 %	9,03 %	13,83 %	18,69 %	23,57 %	28,48 %	33,42 %	38,37 %	43,34 %	48,33 %	53,33 %	58,35 %	63,39 %	68,45 %	73,53 %	78,64 %	83,77 %	88,96 %	94,22 %	99,86 %

	Rate																				
3600	0,70 %	4,31 %	9,04 %	13,85 %	18,70 %	23,59 %	28,51 %	33,44 %	38,39 %	43,37 %	48,35 %	53,36 %	58,38 %	63,42 %	68,47 %	73,55 %	78,65 %	83,79 %	88,97 %	94,24 %	99,86 %
3700	0,71 %	4,32 %	9,05 %	13,86 %	18,72 %	23,61 %	28,53 %	33,46 %	38,42 %	43,39 %	48,38 %	53,38 %	58,40 %	63,44 %	68,49 %	73,57 %	78,67 %	83,81 %	88,99 %	94,25 %	99,86 %
3800	0,71 %	4,33 %	9,06 %	13,88 %	18,74 %	23,63 %	28,55 %	33,48 %	38,44 %	43,41 %	48,40 %	53,40 %	58,42 %	63,46 %	68,51 %	73,59 %	78,69 %	83,82 %	89,00 %	94,26 %	99,87 %
3900	0,71 %	4,34 %	9,08 %	13,89 %	18,75 %	23,65 %	28,56 %	33,50 %	38,46 %	43,43 %	48,42 %	53,42 %	58,44 %	63,48 %	68,53 %	73,61 %	78,71 %	83,84 %	89,01 %	94,27 %	99,87 %
4000	0,72 %	4,35 %	9,09 %	13,91 %	18,77 %	23,66 %	28,58 %	33,52 %	38,48 %	43,45 %	48,44 %	53,44 %	58,46 %	63,50 %	68,55 %	73,63 %	78,73 %	83,86 %	89,03 %	94,28 %	99,87 %
4100	0,72 %	4,35 %	9,10 %	13,92 %	18,79 %	23,68 %	28,60 %	33,54 %	38,50 %	43,47 %	48,46 %	53,46 %	58,48 %	63,52 %	68,57 %	73,64 %	78,74 %	83,87 %	89,04 %	94,29 %	99,88 %
4200	0,72 %	4,36 %	9,11 %	13,93 %	18,80 %	23,70 %	28,62 %	33,56 %	38,51 %	43,49 %	48,48 %	53,48 %	58,50 %	63,54 %	68,59 %	73,66 %	78,76 %	83,88 %	89,05 %	94,30 %	99,88 %
4300	0,72 %	4,37 %	9,12 %	13,95 %	18,81 %	23,71 %	28,63 %	33,57 %	38,53 %	43,51 %	48,49 %	53,50 %	58,52 %	63,55 %	68,61 %	73,68 %	78,77 %	83,90 %	89,06 %	94,30 %	99,88 %
4400	0,73 %	4,37 %	9,13 %	13,96 %	18,83 %	23,73 %	28,65 %	33,59 %	38,55 %	43,52 %	48,51 %	53,52 %	58,53 %	63,57 %	68,62 %	73,69 %	78,79 %	83,91 %	89,08 %	94,31 %	99,89 %
4500	0,73 %	4,38 %	9,14 %	13,97 %	18,84 %	23,74 %	28,66 %	33,61 %	38,56 %	43,54 %	48,53 %	53,53 %	58,55 %	63,59 %	68,64 %	73,71 %	78,80 %	83,92 %	89,09 %	94,32 %	99,89 %
4600	0,73 %	4,39 %	9,15 %	13,98 %	18,85 %	23,75 %	28,68 %	33,62 %	38,58 %	43,56 %	48,54 %	53,55 %	58,57 %	63,60 %	68,65 %	73,72 %	78,81 %	83,93 %	89,10 %	94,33 %	99,89 %
4700	0,74 %	4,39 %	9,16 %	13,99 %	18,86 %	23,77 %	28,69 %	33,64 %	38,60 %	43,57 %	48,56 %	53,56 %	58,58 %	63,62 %	68,67 %	73,74 %	78,83 %	83,95 %	89,11 %	94,34 %	99,89 %
4800	0,74 %	4,40 %	9,17 %	14,00 %	18,88 %	23,78 %	28,71 %	33,65 %	38,61 %	43,59 %	48,58 %	53,58 %	58,60 %	63,63 %	68,68 %	73,75 %	78,84 %	83,96 %	89,12 %	94,34 %	99,90 %
4900	0,74 %	4,41 %	9,17 %	14,01 %	18,89 %	23,79 %	28,72 %	33,66 %	38,62 %	43,60 %	48,59 %	53,59 %	58,61 %	63,65 %	68,69 %	73,76 %	78,85 %	83,97 %	89,13 %	94,35 %	99,90 %
5000	0,74 %	4,41 %	9,18 %	14,02 %	18,90 %	23,80 %	28,73 %	33,68 %	38,64 %	43,61 %	48,60 %	53,61 %	58,63 %	63,66 %	68,71 %	73,78 %	78,86 %	83,98 %	89,13 %	94,36 %	99,90 %

A.4.5 Upper limit of confidence intervals according to Pearson-Clopper formula

Table A.4.2 contains values which specify the upper limit of the confidence interval. The upper limit depends on the number of samples and the according rate value. In figures A.4.1 to A.4.3 this information can be found at the red lines.

Table A.4.2

	Rate																				
NrMeas	1 %	5 %	10 %	15 %	20 %	25 %	30 %	35 %	40 %	45 %	50 %	55 %	60 %	65 %	70 %	75 %	80 %	85 %	90 %	95 %	100 %
100	5,45 %	11,28 %	17,62 %	23,53 %	29,18 %	34,66 %	39,98 %	45,18 %	50,28 %	55,30 %	60,19 %	64,98 %	69,67 %	74,27 %	78,76 %	83,18 %	87,41 %	91,44 %	95,20 %	98,48 %	100,00 %
200	3,57 %	9,00 %	15,02 %	20,72 %	26,22 %	31,60 %	36,87 %	42,05 %	47,15 %	52,18 %	57,14 %	62,03 %	66,85 %	71,59 %	76,26 %	80,86 %	85,34 %	89,68 %	93,82 %	97,63 %	100,00 %
300	2,89 %	8,11 %	13,97 %	19,55 %	24,98 %	30,30 %	35,53 %	40,69 %	45,79 %	50,82 %	55,80 %	60,73 %	65,59 %	70,39 %	75,14 %	79,81 %	84,39 %	88,86 %	93,17 %	97,21 %	100,00 %
400	2,54 %	7,62 %	13,37 %	18,88 %	24,26 %	29,54 %	34,75 %	39,90 %	44,99 %	50,03 %	55,01 %	59,95 %	64,84 %	69,67 %	74,45 %	79,18 %	83,82 %	88,37 %	92,77 %	96,94 %	100,00 %
500	2,32 %	7,29 %	12,97 %	18,44 %	23,78 %	29,04 %	34,23 %	39,36 %	44,45 %	49,48 %	54,47 %	59,42 %	64,32 %	69,18 %	73,99 %	78,74 %	83,43 %	88,02 %	92,50 %	96,75 %	100,00 %
600	2,16 %	7,06 %	12,68 %	18,11 %	23,43 %	28,67 %	33,84 %	38,97 %	44,05 %	49,08 %	54,08 %	59,03 %	63,95 %	68,82 %	73,65 %	78,42 %	83,13 %	87,77 %	92,29 %	96,61 %	100,00 %
700	2,05 %	6,89 %	12,47 %	17,86 %	23,16 %	28,38 %	33,55 %	38,66 %	43,74 %	48,77 %	53,77 %	58,73 %	63,65 %	68,54 %	73,38 %	78,17 %	82,91 %	87,57 %	92,13 %	96,50 %	100,00 %
800	1,96 %	6,75 %	12,29 %	17,67 %	22,94 %	28,15 %	33,31 %	38,42 %	43,49 %	48,52 %	53,52 %	58,49 %	63,42 %	68,31 %	73,16 %	77,97 %	82,72 %	87,41 %	92,00 %	96,41 %	100,00 %
900	1,89 %	6,63 %	12,15 %	17,50 %	22,77 %	27,96 %	33,11 %	38,22 %	43,29 %	48,32 %	53,32 %	58,29 %	63,22 %	68,12 %	72,98 %	77,80 %	82,57 %	87,27 %	91,89 %	96,34 %	100,00 %
1000	1,83 %	6,54 %	12,03 %	17,37 %	22,62 %	27,81 %	32,95 %	38,05 %	43,11 %	48,15 %	53,15 %	58,12 %	63,05 %	67,96 %	72,83 %	77,66 %	82,44 %	87,16 %	91,79 %	96,27 %	100,00 %
1100	1,78 %	6,46 %	11,93 %	17,25 %	22,49 %	27,67 %	32,80 %	37,90 %	42,97 %	48,00 %	53,00 %	57,97 %	62,91 %	67,82 %	72,70 %	77,54 %	82,33 %	87,06 %	91,71 %	96,22 %	100,00 %
1200	1,74 %	6,39 %	11,84 %	17,15 %	22,38 %	27,55 %	32,68 %	37,77 %	42,84 %	47,87 %	52,87 %	57,84 %	62,79 %	67,70 %	72,58 %	77,43 %	82,23 %	86,98 %	91,64 %	96,17 %	100,00 %
1300	1,70 %	6,33 %	11,76 %	17,06 %	22,28 %	27,45 %	32,57 %	37,66 %	42,72 %	47,75 %	52,75 %	57,73 %	62,68 %	67,59 %	72,48 %	77,33 %	82,14 %	86,90 %	91,58 %	96,12 %	100,00 %
1400	1,67 %	6,28 %	11,69 %	16,98 %	22,19 %	27,35 %	32,48 %	37,56 %	42,62 %	47,65 %	52,65 %	57,63 %	62,58 %	67,50 %	72,39 %	77,25 %	82,07 %	86,83 %	91,52 %	96,09 %	100,00 %
1500	1,64 %	6,23 %	11,63 %	16,91 %	22,12 %	27,27 %	32,39 %	37,47 %	42,53 %	47,56 %	52,56 %	57,54 %	62,49 %	67,42 %	72,31 %	77,18 %	82,00 %	86,77 %	91,47 %	96,05 %	100,00 %
1600	1,62 %	6,18 %	11,58 %	16,84 %	22,05 %	27,20 %	32,31 %	37,39 %	42,45 %	47,48 %	52,48 %	57,46 %	62,41 %	67,34 %	72,24 %	77,11 %	81,94 %	86,72 %	91,43 %	96,02 %	100,00 %
1700	1,60 %	6,15 %	11,53 %	16,79 %	21,98 %	27,13 %	32,24 %	37,32 %	42,37 %	47,40 %	52,40 %	57,38 %	62,34 %	67,27 %	72,17 %	77,04 %	81,88 %	86,67 %	91,39 %	95,99 %	100,00 %

	Rate																				
1800	1,58 %	6,11 %	11,48 %	16,73 %	21,92 %	27,07 %	32,18 %	37,25 %	42,31 %	47,33 %	52,34 %	57,32 %	62,27 %	67,21 %	72,11 %	76,99 %	81,83 %	86,62 %	91,35 %	95,96 %	100,00 %
1900	1,56 %	6,08 %	11,44 %	16,69 %	21,87 %	27,01 %	32,12 %	37,19 %	42,24 %	47,27 %	52,27 %	57,25 %	62,21 %	67,15 %	72,06 %	76,93 %	81,78 %	86,58 %	91,31 %	95,94 %	100,00 %
2000	1,54 %	6,05 %	11,40 %	16,64 %	21,82 %	26,96 %	32,06 %	37,14 %	42,19 %	47,21 %	52,22 %	57,20 %	62,16 %	67,09 %	72,00 %	76,89 %	81,73 %	86,54 %	91,28 %	95,92 %	100,00 %
2100	1,52 %	6,02 %	11,36 %	16,60 %	21,78 %	26,91 %	32,01 %	37,08 %	42,13 %	47,16 %	52,16 %	57,14 %	62,10 %	67,04 %	71,96 %	76,84 %	81,69 %	86,50 %	91,25 %	95,89 %	100,00 %
2200	1,51 %	6,00 %	11,33 %	16,56 %	21,73 %	26,86 %	31,96 %	37,03 %	42,08 %	47,11 %	52,11 %	57,09 %	62,06 %	66,99 %	71,91 %	76,80 %	81,65 %	86,47 %	91,22 %	95,87 %	100,00 %
2300	1,50 %	5,97 %	11,30 %	16,53 %	21,69 %	26,82 %	31,92 %	36,99 %	42,04 %	47,06 %	52,06 %	57,05 %	62,01 %	66,95 %	71,87 %	76,76 %	81,62 %	86,44 %	91,20 %	95,86 %	100,00 %
2400	1,48 %	5,95 %	11,27 %	16,49 %	21,66 %	26,78 %	31,88 %	36,95 %	41,99 %	47,02 %	52,02 %	57,00 %	61,97 %	66,91 %	71,83 %	76,72 %	81,58 %	86,41 %	91,17 %	95,84 %	100,00 %
2500	1,47 %	5,93 %	11,24 %	16,46 %	21,62 %	26,75 %	31,84 %	36,91 %	41,95 %	46,98 %	51,98 %	56,96 %	61,93 %	66,87 %	71,79 %	76,69 %	81,55 %	86,38 %	91,15 %	95,82 %	100,00 %
2600	1,46 %	5,91 %	11,22 %	16,43 %	21,59 %	26,71 %	31,80 %	36,87 %	41,91 %	46,94 %	51,94 %	56,93 %	61,89 %	66,83 %	71,76 %	76,66 %	81,52 %	86,35 %	91,13 %	95,81 %	100,00 %
2700	1,45 %	5,89 %	11,19 %	16,40 %	21,56 %	26,68 %	31,77 %	36,83 %	41,88 %	46,90 %	51,90 %	56,89 %	61,85 %	66,80 %	71,72 %	76,62 %	81,49 %	86,33 %	91,11 %	95,79 %	100,00 %
2800	1,44 %	5,87 %	11,17 %	16,38 %	21,53 %	26,65 %	31,74 %	36,80 %	41,84 %	46,87 %	51,87 %	56,85 %	61,82 %	66,77 %	71,69 %	76,60 %	81,47 %	86,30 %	91,09 %	95,78 %	100,00 %
2900	1,43 %	5,86 %	11,15 %	16,35 %	21,50 %	26,62 %	31,70 %	36,77 %	41,81 %	46,83 %	51,84 %	56,82 %	61,79 %	66,74 %	71,66 %	76,57 %	81,44 %	86,28 %	91,07 %	95,77 %	100,00 %
3000	1,42 %	5,84 %	11,13 %	16,33 %	21,48 %	26,59 %	31,68 %	36,74 %	41,78 %	46,80 %	51,81 %	56,79 %	61,76 %	66,71 %	71,64 %	76,54 %	81,42 %	86,26 %	91,05 %	95,75 %	100,00 %
3100	1,42 %	5,83 %	11,11 %	16,31 %	21,45 %	26,56 %	31,65 %	36,71 %	41,75 %	46,77 %	51,78 %	56,76 %	61,73 %	66,68 %	71,61 %	76,52 %	81,40 %	86,24 %	91,03 %	95,74 %	100,00 %
3200	1,41 %	5,81 %	11,09 %	16,28 %	21,43 %	26,54 %	31,62 %	36,68 %	41,72 %	46,74 %	51,75 %	56,73 %	61,70 %	66,65 %	71,58 %	76,49 %	81,37 %	86,22 %	91,02 %	95,73 %	100,00 %
3300	1,40 %	5,80 %	11,07 %	16,26 %	21,41 %	26,51 %	31,60 %	36,66 %	41,69 %	46,72 %	51,72 %	56,71 %	61,68 %	66,63 %	71,56 %	76,47 %	81,35 %	86,20 %	91,00 %	95,72 %	100,00 %
3400	1,39 %	5,79 %	11,06 %	16,24 %	21,38 %	26,49 %	31,57 %	36,63 %	41,67 %	46,69 %	51,69 %	56,68 %	61,65 %	66,60 %	71,54 %	76,45 %	81,33 %	86,18 %	90,99 %	95,71 %	100,00 %
3500	1,39 %	5,77 %	11,04 %	16,23 %	21,36 %	26,47 %	31,55 %	36,61 %	41,65 %	46,67 %	51,67 %	56,66 %	61,63 %	66,58 %	71,52 %	76,43 %	81,31 %	86,17 %	90,98 %	95,70 %	100,00 %
3600	1,38 %	5,76 %	11,03 %	16,21 %	21,34 %	26,45 %	31,53 %	36,58 %	41,62 %	46,64 %	51,65 %	56,63 %	61,61 %	66,56 %	71,49 %	76,41 %	81,30 %	86,15 %	90,96 %	95,69 %	100,00 %
3700	1,38 %	5,75 %	11,01 %	16,19 %	21,33 %	26,43 %	31,51 %	36,56 %	41,60 %	46,62 %	51,62 %	56,61 %	61,58 %	66,54 %	71,47 %	76,39 %	81,28 %	86,14 %	90,95 %	95,68 %	100,00 %
3800	1,37 %	5,74 %	11,00 %	16,18 %	21,31 %	26,41 %	31,49 %	36,54 %	41,58 %	46,60 %	51,60 %	56,59 %	61,56 %	66,52 %	71,45 %	76,37 %	81,26 %	86,12 %	90,94 %	95,67 %	100,00 %
3900	1,36 %	5,73 %	10,98 %	16,16 %	21,29 %	26,39 %	31,47 %	36,52 %	41,56 %	46,58 %	51,58 %	56,57 %	61,54 %	66,50 %	71,44 %	76,35 %	81,25 %	86,11 %	90,92 %	95,66 %	100,00 %
4000	1,36 %	5,72 %	10,97 %	16,14 %	21,27 %	26,37 %	31,45 %	36,50 %	41,54 %	46,56 %	51,56 %	56,55 %	61,52 %	66,48 %	71,42 %	76,34 %	81,23 %	86,09 %	90,91 %	95,66 %	100,00 %
4100	1,35 %	5,71 %	10,96 %	16,13 %	21,26 %	26,36 %	31,43 %	36,48 %	41,52 %	46,54 %	51,54 %	56,53 %	61,50 %	66,46 %	71,40 %	76,32 %	81,22 %	86,08 %	90,90 %	95,65 %	100,00 %
4200	1,35 %	5,70 %	10,95 %	16,12 %	21,24 %	26,34 %	31,41 %	36,46 %	41,50 %	46,52 %	51,52 %	56,51 %	61,49 %	66,44 %	71,38 %	76,30 %	81,20 %	86,07 %	90,89 %	95,64 %	100,00 %
4300	1,34 %	5,69 %	10,94 %	16,10 %	21,23 %	26,32 %	31,39 %	36,45 %	41,48 %	46,50 %	51,51 %	56,49 %	61,47 %	66,43 %	71,37 %	76,29 %	81,19 %	86,06 %	90,88 %	95,63 %	100,00 %
4400	1,34 %	5,69 %	10,92 %	16,09 %	21,21 %	26,31 %	31,38 %	36,43 %	41,47 %	46,48 %	51,49 %	56,48 %	61,45 %	66,41 %	71,35 %	76,27 %	81,17 %	86,04 %	90,87 %	95,63 %	100,00 %
4500	1,34 %	5,68 %	10,91 %	16,08 %	21,20 %	26,29 %	31,36 %	36,41 %	41,45 %	46,47 %	51,47 %	56,46 %	61,44 %	66,39 %	71,34 %	76,26 %	81,16 %	86,03 %	90,86 %	95,62 %	100,00 %
4600	1,33 %	5,67 %	10,90 %	16,06 %	21,19 %	26,28 %	31,35 %	36,40 %	41,43 %	46,45 %	51,46 %	56,45 %	61,42 %	66,38 %	71,32 %	76,25 %	81,15 %	86,02 %	90,85 %	95,61 %	100,00 %
4700	1,33 %	5,66 %	10,89 %	16,05 %	21,17 %	26,26 %	31,33 %	36,38 %	41,42 %	46,44 %	51,44 %	56,43 %	61,40 %	66,36 %	71,31 %	76,23 %	81,14 %	86,01 %	90,84 %	95,61 %	100,00 %
4800	1,32 %	5,65 %	10,88 %	16,04 %	21,16 %	26,25 %	31,32 %	36,37 %	41,40 %	46,42 %	51,42 %	56,41 %	61,39 %	66,35 %	71,29 %	76,22 %	81,12 %	86,00 %	90,83 %	95,60 %	100,00 %
4900	1,32 %	5,65 %	10,87 %	16,03 %	21,15 %	26,24 %	31,30 %	36,35 %	41,39 %	46,41 %	51,41 %	56,40 %	61,38 %	66,34 %	71,28 %	76,21 %	81,11 %	85,99 %	90,83 %	95,59 %	100,00 %
5000	1,32 %	5,64 %	10,87 %	16,02 %	21,14 %	26,22 %	31,29 %	36,34 %	41,37 %	46,39 %	51,40 %	56,39 %	61,36 %	66,32 %	71,27 %	76,20 %	81,10 %	85,98 %	90,82 %	95,59 %	100,00 %

A.4.6 Span of confidence intervals according to Pearson-Clopper formula

Table A.4.3 contains values which specify the difference ("span") between the upper and the lower limit of the confidence interval. The span depends on the number of samples and the according rate value. In figures A.4.1 to A.4.3 this information can be found as the vertical distance between the red and the blue lines.

Table A.4.3

	Rate																				
NrMeas	1 %	5 %	10 %	15 %	20 %	25 %	30 %	35 %	40 %	45 %	50 %	55 %	60 %	65 %	70 %	75 %	80 %	85 %	90 %	95 %	100 %
100	5,42 %	9,64 %	12,72 %	14,89 %	16,52 %	17,78 %	18,74 %	19,46 %	19,95 %	20,30 %	20,38 %	20,28 %	19,95 %	19,46 %	18,74 %	17,90 %	16,67 %	15,07 %	12,96 %	10,00 %	4,85 %
200	3,44 %	6,58 %	8,81 %	10,36 %	11,53 %	12,44 %	13,13 %	13,64 %	13,99 %	14,22 %	14,29 %	14,21 %	13,99 %	13,64 %	13,13 %	12,48 %	11,59 %	10,43 %	8,90 %	6,71 %	2,47 %
300	2,69 %	5,29 %	7,12 %	8,40 %	9,36 %	10,10 %	10,66 %	11,08 %	11,39 %	11,54 %	11,61 %	11,55 %	11,37 %	11,08 %	10,68 %	10,12 %	9,39 %	8,43 %	7,17 %	5,36 %	1,66 %
400	2,27 %	4,54 %	6,13 %	7,24 %	8,07 %	8,71 %	9,21 %	9,57 %	9,82 %	9,98 %	10,02 %	9,97 %	9,82 %	9,57 %	9,21 %	8,73 %	8,09 %	7,26 %	6,16 %	4,59 %	1,25 %
500	1,99 %	4,03 %	5,46 %	6,45 %	7,20 %	7,77 %	8,21 %	8,54 %	8,77 %	8,90 %	8,95 %	8,90 %	8,77 %	8,54 %	8,22 %	7,79 %	7,21 %	6,47 %	5,48 %	4,07 %	1,00 %
600	1,80 %	3,66 %	4,97 %	5,87 %	6,56 %	7,08 %	7,49 %	7,78 %	8,00 %	8,11 %	8,15 %	8,11 %	7,99 %	7,78 %	7,49 %	7,09 %	6,57 %	5,89 %	4,98 %	3,69 %	0,83 %
700	1,65 %	3,38 %	4,59 %	5,43 %	6,06 %	6,55 %	6,92 %	7,20 %	7,39 %	7,50 %	7,54 %	7,50 %	7,39 %	7,20 %	6,93 %	6,56 %	6,07 %	5,44 %	4,60 %	3,40 %	0,71 %
800	1,53 %	3,15 %	4,28 %	5,07 %	5,66 %	6,12 %	6,47 %	6,73 %	6,90 %	7,01 %	7,05 %	7,01 %	6,90 %	6,73 %	6,47 %	6,12 %	5,67 %	5,08 %	4,29 %	3,17 %	0,63 %
900	1,43 %	2,96 %	4,03 %	4,77 %	5,33 %	5,76 %	6,09 %	6,34 %	6,51 %	6,60 %	6,64 %	6,60 %	6,50 %	6,34 %	6,10 %	5,77 %	5,34 %	4,78 %	4,04 %	2,98 %	0,56 %
1000	1,35 %	2,81 %	3,82 %	4,52 %	5,05 %	5,46 %	5,77 %	6,01 %	6,17 %	6,26 %	6,29 %	6,26 %	6,17 %	6,01 %	5,78 %	5,47 %	5,06 %	4,53 %	3,83 %	2,82 %	0,50 %
1100	1,28 %	2,67 %	3,64 %	4,31 %	4,81 %	5,20 %	5,50 %	5,72 %	5,88 %	5,97 %	6,00 %	5,97 %	5,87 %	5,72 %	5,50 %	5,21 %	4,82 %	4,31 %	3,64 %	2,68 %	0,46 %
1200	1,22 %	2,55 %	3,48 %	4,12 %	4,61 %	4,98 %	5,26 %	5,48 %	5,62 %	5,71 %	5,74 %	5,71 %	5,62 %	5,48 %	5,27 %	4,98 %	4,61 %	4,13 %	3,48 %	2,56 %	0,42 %
1300	1,17 %	2,45 %	3,34 %	3,96 %	4,42 %	4,78 %	5,05 %	5,26 %	5,40 %	5,48 %	5,51 %	5,48 %	5,40 %	5,26 %	5,06 %	4,78 %	4,43 %	3,96 %	3,34 %	2,46 %	0,39 %
1400	1,12 %	2,36 %	3,21 %	3,81 %	4,26 %	4,60 %	4,87 %	5,07 %	5,20 %	5,28 %	5,31 %	5,28 %	5,20 %	5,06 %	4,87 %	4,61 %	4,26 %	3,81 %	3,22 %	2,36 %	0,36 %
1500	1,08 %	2,27 %	3,10 %	3,68 %	4,11 %	4,45 %	4,70 %	4,89 %	5,02 %	5,10 %	5,12 %	5,10 %	5,02 %	4,89 %	4,70 %	4,45 %	4,12 %	3,68 %	3,11 %	2,28 %	0,33 %
1600	1,05 %	2,20 %	3,00 %	3,56 %	3,98 %	4,30 %	4,55 %	4,73 %	4,86 %	4,93 %	4,96 %	4,93 %	4,86 %	4,73 %	4,55 %	4,31 %	3,98 %	3,56 %	3,01 %	2,21 %	0,31 %
1700	1,01 %	2,13 %	2,91 %	3,45 %	3,86 %	4,17 %	4,41 %	4,59 %	4,71 %	4,79 %	4,81 %	4,79 %	4,71 %	4,59 %	4,41 %	4,17 %	3,86 %	3,46 %	2,91 %	2,14 %	0,29 %
1800	0,98 %	2,07 %	2,83 %	3,35 %	3,75 %	4,05 %	4,29 %	4,46 %	4,58 %	4,65 %	4,67 %	4,65 %	4,58 %	4,46 %	4,29 %	4,06 %	3,75 %	3,36 %	2,83 %	2,08 %	0,28 %
1900	0,95 %	2,01 %	2,75 %	3,26 %	3,65 %	3,94 %	4,17 %	4,34 %	4,46 %	4,52 %	4,55 %	4,52 %	4,46 %	4,34 %	4,17 %	3,95 %	3,65 %	3,27 %	2,75 %	2,02 %	0,26 %
2000	0,93 %	1,96 %	2,68 %	3,18 %	3,55 %	3,84 %	4,06 %	4,23 %	4,34 %	4,41 %	4,43 %	4,41 %	4,34 %	4,23 %	4,07 %	3,84 %	3,56 %	3,18 %	2,68 %	1,97 %	0,25 %
2100	0,90 %	1,91 %	2,61 %	3,10 %	3,47 %	3,75 %	3,97 %	4,13 %	4,24 %	4,30 %	4,32 %	4,30 %	4,24 %	4,13 %	3,97 %	3,75 %	3,47 %	3,10 %	2,62 %	1,92 %	0,24 %
2200	0,88 %	1,87 %	2,55 %	3,03 %	3,39 %	3,66 %	3,87 %	4,03 %	4,14 %	4,20 %	4,22 %	4,20 %	4,14 %	4,03 %	3,87 %	3,66 %	3,39 %	3,03 %	2,56 %	1,87 %	0,23 %
2300	0,86 %	1,83 %	2,50 %	2,96 %	3,31 %	3,58 %	3,79 %	3,94 %	4,05 %	4,11 %	4,13 %	4,11 %	4,05 %	3,94 %	3,79 %	3,58 %	3,31 %	2,96 %	2,50 %	1,83 %	0,22 %
2400	0,84 %	1,79 %	2,44 %	2,90 %	3,24 %	3,50 %	3,71 %	3,86 %	3,96 %	4,02 %	4,04 %	4,02 %	3,96 %	3,86 %	3,71 %	3,51 %	3,24 %	2,90 %	2,44 %	1,79 %	0,21 %
2500	0,82 %	1,75 %	2,39 %	2,84 %	3,17 %	3,43 %	3,63 %	3,78 %	3,88 %	3,94 %	3,96 %	3,94 %	3,88 %	3,78 %	3,63 %	3,43 %	3,18 %	2,84 %	2,39 %	1,75 %	0,20 %
2600	0,81 %	1,71 %	2,34 %	2,78 %	3,11 %	3,37 %	3,56 %	3,70 %	3,80 %	3,86 %	3,88 %	3,86 %	3,80 %	3,70 %	3,56 %	3,37 %	3,11 %	2,78 %	2,35 %	1,72 %	0,19 %
2700	0,79 %	1,68 %	2,30 %	2,73 %	3,05 %	3,30 %	3,49 %	3,63 %	3,73 %	3,79 %	3,81 %	3,79 %	3,73 %	3,63 %	3,49 %	3,30 %	3,05 %	2,73 %	2,30 %	1,68 %	0,19 %
2800	0,78 %	1,65 %	2,26 %	2,68 %	3,00 %	3,24 %	3,43 %	3,57 %	3,66 %	3,72 %	3,74 %	3,72 %	3,66 %	3,57 %	3,43 %	3,24 %	3,00 %	2,68 %	2,26 %	1,65 %	0,18 %
2900	0,76 %	1,62 %	2,22 %	2,63 %	2,95 %	3,19 %	3,37 %	3,51 %	3,60 %	3,65 %	3,67 %	3,65 %	3,60 %	3,50 %	3,37 %	3,19 %	2,95 %	2,63 %	2,22 %	1,62 %	0,17 %
3000	0,75 %	1,59 %	2,18 %	2,59 %	2,90 %	3,13 %	3,31 %	3,45 %	3,54 %	3,59 %	3,61 %	3,59 %	3,54 %	3,45 %	3,31 %	3,13 %	2,90 %	2,59 %	2,18 %	1,60 %	0,17 %
3100	0,74 %	1,57 %	2,14 %	2,55 %	2,85 %	3,08 %	3,26 %	3,39 %	3,48 %	3,53 %	3,55 %	3,53 %	3,48 %	3,39 %	3,26 %	3,08 %	2,85 %	2,55 %	2,15 %	1,57 %	0,16 %
3200	0,72 %	1,54 %	2,11 %	2,51 %	2,80 %	3,03 %	3,21 %	3,34 %	3,42 %	3,48 %	3,49 %	3,48 %	3,42 %	3,34 %	3,21 %	3,03 %	2,80 %	2,51 %	2,11 %	1,54 %	0,16 %
3300	0,71 %	1,52 %	2,08 %	2,47 %	2,76 %	2,98 %	3,16 %	3,28 %	3,37 %	3,42 %	3,44 %	3,42 %	3,37 %	3,28 %	3,16 %	2,98 %	2,76 %	2,47 %	2,08 %	1,52 %	0,15 %
3400	0,70 %	1,50 %	2,05 %	2,43 %	2,72 %	2,94 %	3,11 %	3,23 %	3,32 %	3,37 %	3,39 %	3,37 %	3,32 %	3,23 %	3,11 %	2,94 %	2,72 %	2,43 %	2,05 %	1,50 %	0,15 %
3500	0,69 %	1,47 %	2,02 %	2,39 %	2,68 %	2,90 %	3,06 %	3,19 %	3,27 %	3,32 %	3,34 %	3,32 %	3,27 %	3,19 %	3,06 %	2,90 %	2,68 %	2,40 %	2,02 %	1,48 %	0,14 %
3600	0,68 %	1,45 %	1,99 %	2,36 %	2,64 %	2,86 %	3,02 %	3,14 %	3,23 %	3,28 %	3,29 %	3,28 %	3,23 %	3,14 %	3,02 %	2,86 %	2,64 %	2,36 %	1,99 %	1,45 %	0,14 %

	Rate																				
3700	0,67 %	1,43 %	1,96 %	2,33 %	2,60 %	2,82 %	2,98 %	3,10 %	3,18 %	3,23 %	3,25 %	3,23 %	3,18 %	3,10 %	2,98 %	2,82 %	2,60 %	2,33 %	1,96 %	1,43 %	0,14 %
3800	0,66 %	1,41 %	1,93 %	2,30 %	2,57 %	2,78 %	2,94 %	3,06 %	3,14 %	3,19 %	3,20 %	3,19 %	3,14 %	3,06 %	2,94 %	2,78 %	2,57 %	2,30 %	1,94 %	1,41 %	0,13 %
3900	0,65 %	1,39 %	1,91 %	2,27 %	2,54 %	2,74 %	2,90 %	3,02 %	3,10 %	3,15 %	3,16 %	3,15 %	3,10 %	3,02 %	2,90 %	2,74 %	2,54 %	2,27 %	1,91 %	1,40 %	0,13 %
4000	0,64 %	1,38 %	1,88 %	2,24 %	2,50 %	2,71 %	2,86 %	2,98 %	3,06 %	3,11 %	3,12 %	3,11 %	3,06 %	2,98 %	2,86 %	2,71 %	2,50 %	2,24 %	1,89 %	1,38 %	0,13 %
4100	0,64 %	1,36 %	1,86 %	2,21 %	2,47 %	2,67 %	2,83 %	2,94 %	3,02 %	3,07 %	3,08 %	3,07 %	3,02 %	2,94 %	2,83 %	2,68 %	2,47 %	2,21 %	1,86 %	1,36 %	0,12 %
4200	0,63 %	1,34 %	1,84 %	2,18 %	2,44 %	2,64 %	2,79 %	2,91 %	2,99 %	3,03 %	3,05 %	3,03 %	2,99 %	2,91 %	2,80 %	2,64 %	2,44 %	2,18 %	1,84 %	1,34 %	0,12 %
4300	0,62 %	1,33 %	1,82 %	2,16 %	2,41 %	2,61 %	2,76 %	2,87 %	2,95 %	3,00 %	3,01 %	3,00 %	2,95 %	2,87 %	2,76 %	2,61 %	2,41 %	2,16 %	1,82 %	1,33 %	0,12 %
4400	0,61 %	1,31 %	1,80 %	2,13 %	2,39 %	2,58 %	2,73 %	2,84 %	2,92 %	2,96 %	2,98 %	2,96 %	2,92 %	2,84 %	2,73 %	2,58 %	2,39 %	2,13 %	1,80 %	1,31 %	0,11 %
4500	0,61 %	1,30 %	1,78 %	2,11 %	2,36 %	2,55 %	2,70 %	2,81 %	2,88 %	2,93 %	2,94 %	2,93 %	2,88 %	2,81 %	2,70 %	2,55 %	2,36 %	2,11 %	1,78 %	1,30 %	0,11 %
4600	0,60 %	1,28 %	1,76 %	2,09 %	2,33 %	2,52 %	2,67 %	2,78 %	2,85 %	2,90 %	2,91 %	2,90 %	2,85 %	2,78 %	2,67 %	2,52 %	2,33 %	2,09 %	1,76 %	1,28 %	0,11 %
4700	0,59 %	1,27 %	1,74 %	2,06 %	2,31 %	2,50 %	2,64 %	2,75 %	2,82 %	2,87 %	2,88 %	2,87 %	2,82 %	2,75 %	2,64 %	2,50 %	2,31 %	2,06 %	1,74 %	1,27 %	0,11 %
4800	0,59 %	1,25 %	1,72 %	2,04 %	2,28 %	2,47 %	2,61 %	2,72 %	2,79 %	2,83 %	2,85 %	2,84 %	2,79 %	2,72 %	2,61 %	2,47 %	2,28 %	2,04 %	1,72 %	1,26 %	0,10 %
4900	0,58 %	1,24 %	1,70 %	2,02 %	2,26 %	2,44 %	2,59 %	2,69 %	2,76 %	2,81 %	2,82 %	2,81 %	2,76 %	2,69 %	2,59 %	2,45 %	2,26 %	2,02 %	1,70 %	1,24 %	0,10 %
5000	0,57 %	1,23 %	1,68 %	2,00 %	2,24 %	2,42 %	2,56 %	2,66 %	2,74 %	2,78 %	2,79 %	2,78 %	2,74 %	2,66 %	2,56 %	2,42 %	2,24 %	2,00 %	1,68 %	1,23 %	0,10 %

A.5 Different sample sizes

The following examples show the effect of different sample sizes in a measurement campaign. It is also based on the Pearson-Clopper formulas for the calculation of confidence intervals. Therefore, the examples are valid in a generic way and even for small sample sizes. For higher sample numbers, the calculation of confidence intervals based on the approximation of a normal distribution can be applied.

Three different graphs are depicted: Sample sizes in the range:

- between 100 and 1 100 samples;
- between 1 100 and 2 100 samples; and
- between 1 000 and 11 000 samples.

The depicted curves can be found in the rows of the tables given above (number of measurements is constant, estimated rate varies).

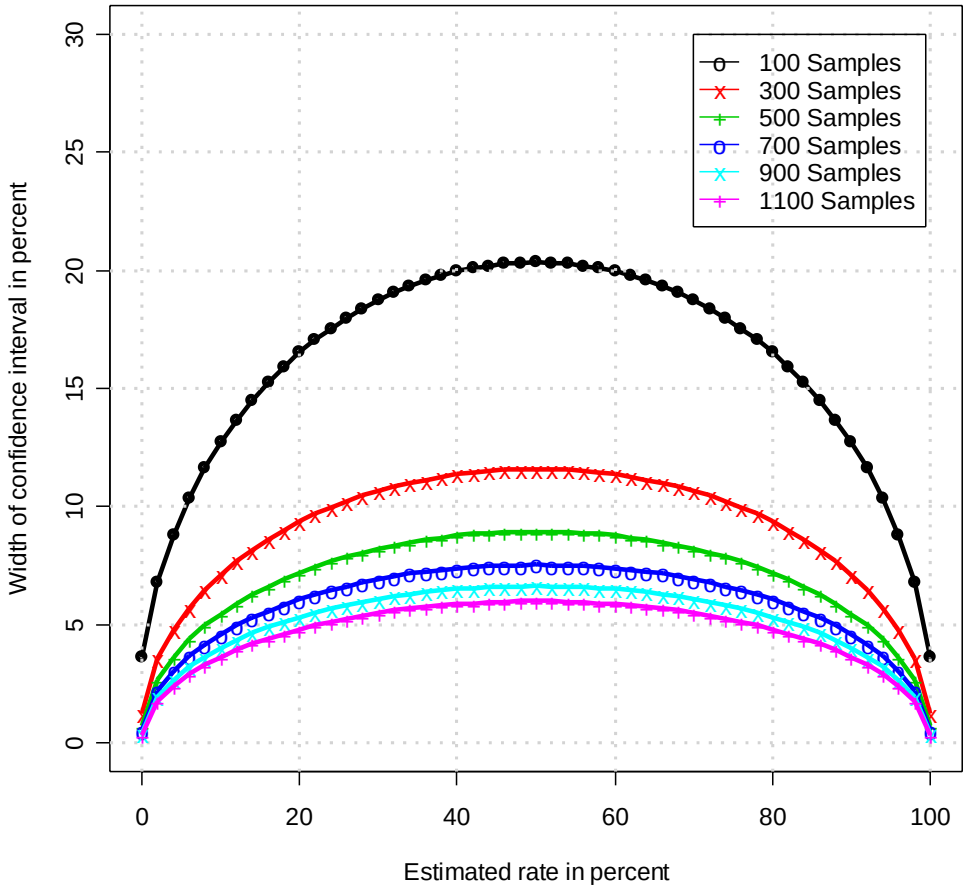


Figure A.5.1: Width of confidence interval for different sample sizes

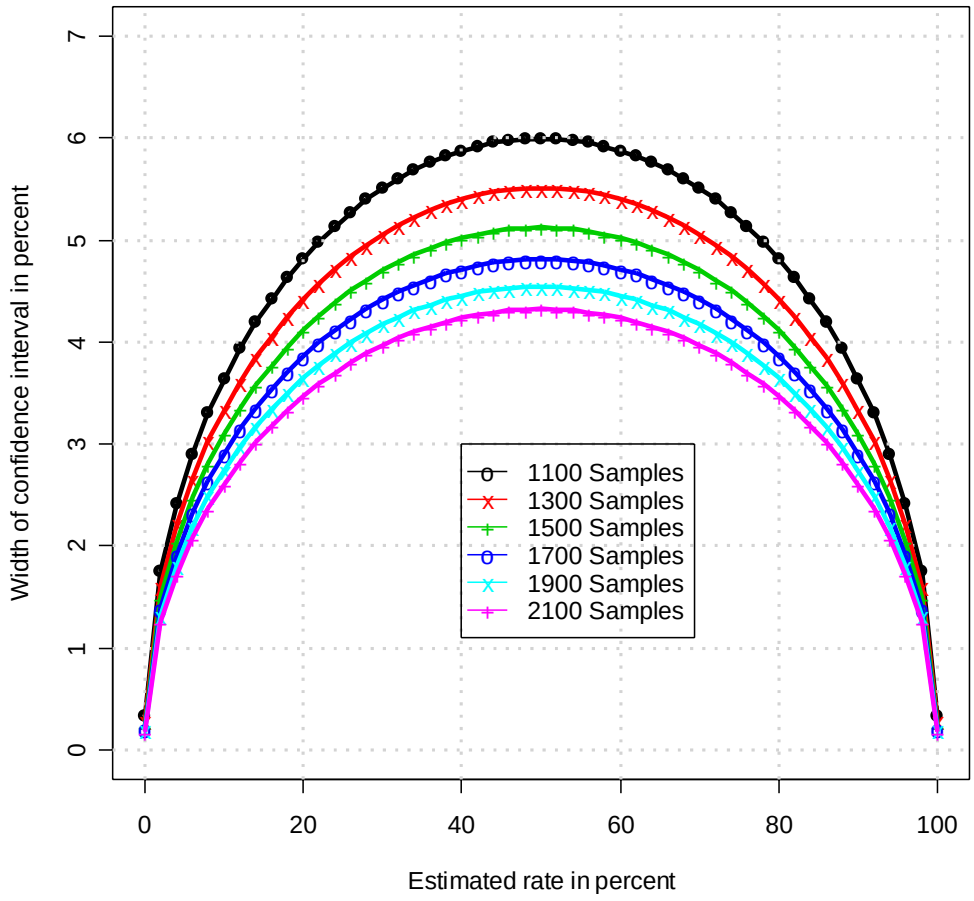


Figure A.5.2: Width of confidence interval for different sample sizes

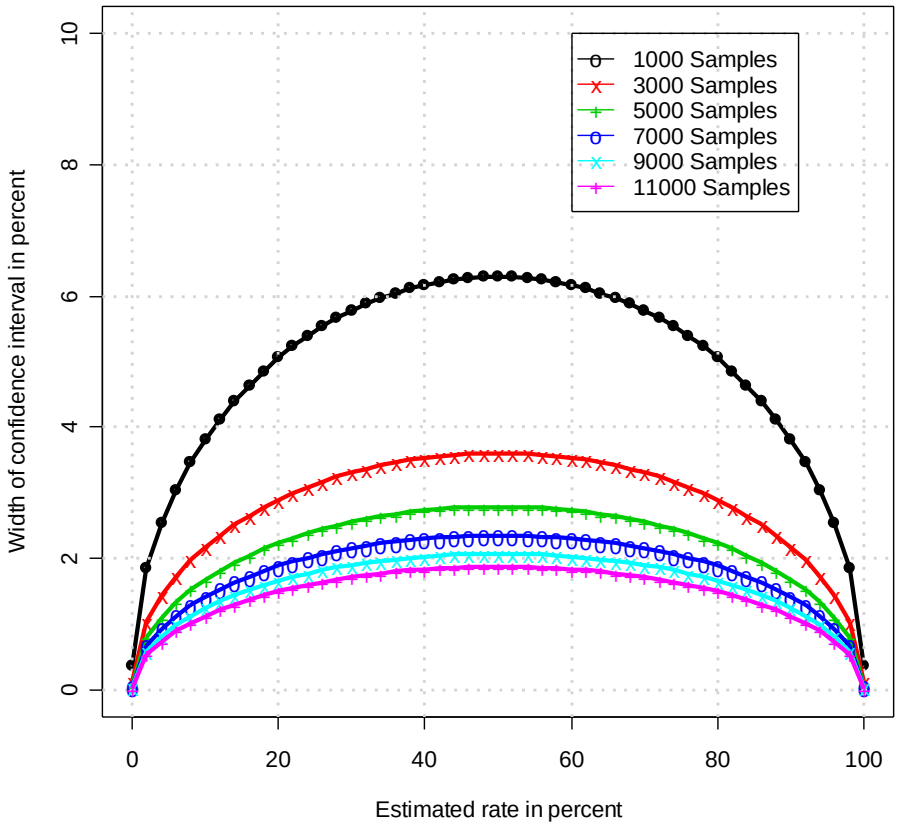


Figure A.5.3: Width of confidence interval for different sample sizes

A.6 Calculation methods

This clause depicts some examples of how to calculate statistical values out of measurement data.

A.6.1 Calculation of quantiles

In this clause the different basic steps to calculate quantile values related to measurement samples are described.

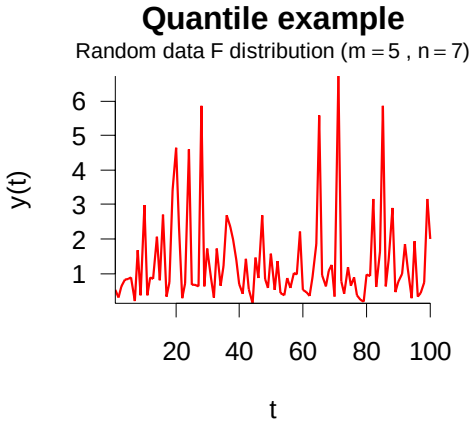


Figure A.6.1: Example of measured data as a time series

Assuming that measurement data according to the graph above has been collected, the following steps can be executed:

- Determine the number *N* of available measurements.
- Sorting of data: The samples are sorted in an ascending order.
- Define the p-quantile value that should be retrieved. In this example, the 95 % quantile (Q95) is requested, so p = 95 % = 0,95.
- Start counting the sorted samples, until you reach the p-percentage of all available samples. In this example, this means 95 % of the samples have to be counted.
- The sample where the according percentage is reached is taken. The appropriate ordinate value represents the searched p-quantile, in this case the 95 % quantile.

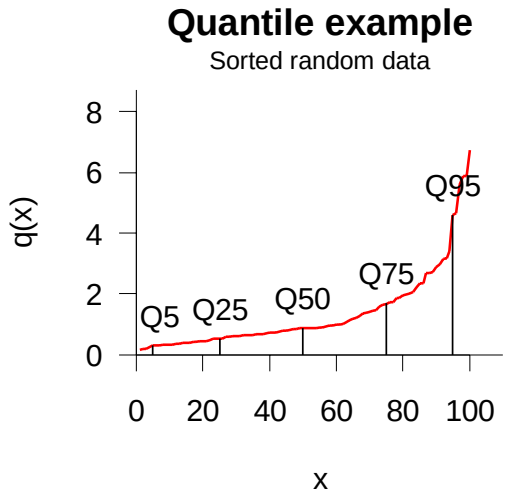


Figure A.6.2: Determination of quantiles on sorted data

The different steps are visualized in the graph above. Further example for other p-quantiles are:

p percentage	5 %	25 %	50 %	75 %	95 %
p-quantile	0,2959737	0,5370118	0,8579087	1,6867595	4,5992459

If for example the 95 % value is not covered by a sample, an interpolation between the left-hand and the right-hand neighbour may be appropriate. This interpolation may have different grades, e.g. linear or quadratic interpolation.

Another possibility to determine quantile values is given by analysis of the Cumulative Distribution Function (CDF). The steps to create a CDF out of measurement results are generally the same as described above.

A.7 Reporting of results

This clause describes which pieces of information should be given to the reader when generating a test report. The categories of different data types are related to the definitions in clause 4.

A.7.1 Methods to use

The variables *x*, *y* and *z* in the following table must be accordingly replaced by the estimated data.

When quantile values are used, it should be kept in mind that the computation of quantiles separates a low percentage of outlier data from the remaining data. This means:

- If lower values represent a better outcome from the customer's perspective, a small percentage containing the highest values could be separated by calculating a 95 %-quantile or a 90 %-quantile. This is the case for example for duration values.
- If higher values represent a better outcome from the customer's perspective, a small percentage containing the lowest values could be separated by calculating a 5 %-quantile or a 10 %-quantile. This is the case for example for throughput values.
- Related to content quality, the appropriate quantile computation orientates itself on the scale of the determined test results. In practice, some algorithms define a value of 0 on a scale from 0 to 5 as the best quality whereas others define the value of 5 as the highest possible quality. The table below gives some hints how to use the quantile computation in these cases.

Table A.7.1

Category of data	Type of information	Method to use	Reporting statement	Additional information	Related clauses
Binary values (Success rates, error rates, ...)	Estimated rate plus confidence interval	Pearson-Clopper	$x\%^{+y_1\%}_{-y_2\%}$	Always valid, borders of confidence interval are asymmetric (except for x = 50)	5.7.2.1
		Gaussian approximation	$x\% \pm y\%$	Applicable if $n \cdot p \cdot q \geq 9$, symmetric borders of confidence interval	5.7.2.2
Duration values (End-to-end delay, establishment delay, ...)	Mean delay plus standard deviation	Empirical mean plus empirical standard deviation	$x\ s \pm y\ s\ (N = z)$	Always valid N: number of samples taken into account	5.4 and 5.5
	α -Quantile plus number of samples	Quantile computation	$q_\alpha = x\ s\ (N = z)$	N: number of samples taken into account α : Desired quantile level, mostly $\alpha = 95\%$ or $\alpha = 90\%$	5.4 and 5.5
Throughput values (Data rates)	Mean data rate plus standard deviation	Empirical mean plus empirical standard deviation	$x\ kbit/s \pm y\ kbit/s\ (N = z)$	Always valid N: number of samples taken into account	5.4 and 5.5
	α -Quantile plus number of samples	Quantile computation	$q_\alpha = x\ kbit/s\ (N = z)$	N: number of samples taken into account α : Desired quantile level, mostly $\alpha = 5\%$ or $\alpha = 10\%$	5.4 and 5.5
Content quality values (Audio quality, video quality)	Mean score plus standard deviation	Empirical mean plus empirical standard deviation	$x\ MOS \pm y\ MOS\ (N = z)$	Always valid N: number of samples taken into account	5.4 and 5.5
	α -Quantile plus number of samples	Quantile computation	$q_\alpha = x\ MOS\ (N = z)$	N: number of samples taken into account α : Desired quantile level, mostly $\alpha = 95\%$ or $\alpha = 90\%$ if lower values represent better quality, $\alpha = 5\%$ or $\alpha = 10\%$ if higher values represent better quality	5.4 and 5.5

A.7.2 Number of significant decimals

When representing final results, the number of reported significant decimals should be orientated on the precision of the evaluation method used (e.g. calculation of standard deviation, confidence interval, ...).

A.7.3 Rounding of end results

During the execution of consecutive calculation steps, no rounding functionality should be applied. Only the final results may be rounded. At least three significant decimals should still remain after applying the rounding functionality whenever possible.

Annex B (informative): Bibliography

[BAT] Bates, D.M. and Chambers, J.M: "Nonlinear regression Analysis and Applications", Wiley & Sons, 1988.

[BRO] I.N. Bronstein, K.A. Semendjajew, G. Musiol, H. Mühlig: "Handbook of Mathematics", Verlag Harri Deutsch; Frankfurt/Main, 1999.

[BOX] Box, Cox: "An analysis of transformations", Journal of the Royal Statistical Society, vol. B 26, p. 211-251, 1964.

[E.800] Terms and Definitions Related to Quality of Service and Network Performance including Dependability, ITU-T Recommendation (1994).

[HARD] W.C. Hardy: "QoS Measurement and Evaluation of Telecommunications Quality of Service", Wiley & Sons, 2001.

[HART] J. Hartung: "Lehr- und Handbuch der angewandten Statistik", Oldenbourg Wissenschaftsverlag, 13th meditation, 2002.

[LAW] A. M. Law, W. D. Kelton: "Simulation modeling and analysis", McGraw-Hill, 3rd edition, 2000.

[MOOD] Mood, Graybill, Boes: "Introduction to the theory of statistics", McCraw-Hill Statistics Series, 1974.

[VEN] Venables, W.N. and Ripley, B.D.: "Modern Applied Statistics with S-Plus", Springer Verlag, 1999.

History

Document history		
V1.1.1	May 2004	Publication
V1.2.1	October 2004	Publication